EECS C128/ ME C134
Midterm (v. 1.01)
Thurs. Mar. 9, 2017
0810-0930 am

Name:_ ________________
SID:___

- Closed book. One page formula sheet. No calculators.

- There are 5 problems worth 100 points total.

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Problem	Points	Score
1	22	
2	22	
3	14	
4	27	
5	15	
TOTAL	100	

Tables for reference:

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1}\frac{2}{3} = 33.7°$	$\tan^{-1}\frac{3}{4} = 36.9°$
$\tan^{-1}1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20\log_{10}1 = 0dB$	$20\log_{10}2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10}5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
	$\sqrt{5} \approx 2.24$	$\sqrt{7} \approx 2.65$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 (22 pts)

Each part is independent.

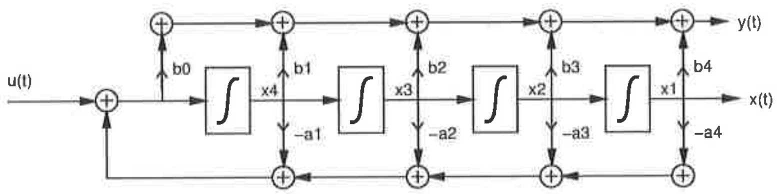
[6 pts] a) Consider a single-input single-output system with transfer function:

$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 4s + 8}{s^2 + 6s + 10}$$

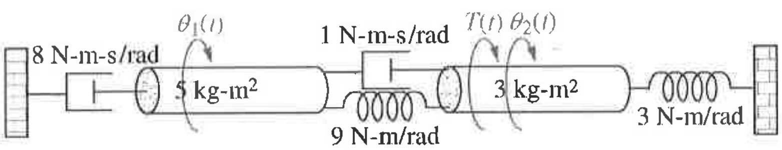
input $u(t)$ and output $y(t)$. Choose coefficients such that the block diagram below has the same transfer function:

$a_1 =$____ $a_2 =$____ $a_3 =$____ $a_4 =$____

$b_0 =$____ $b_1 =$____ $b_2 =$____ $b_3 =$____ $b_4 =$____



[8 pts] b) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to torque and current to rotational velocity. Input torque is $T(t)$.



c) Consider the system

$$G(s) = \frac{s+\beta}{s(s+1)(s+5)}\frac{5}{\beta}$$

[4 pts] i) Find $g(t)$ the inverse Laplace transform of $G(s)$.

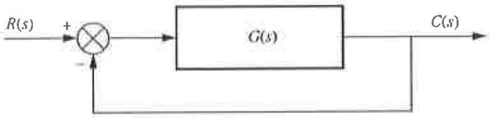
$g(t) = ($ ____ $e^{-t}+$ ____ $e^{-5t}+$ ____ $)u(t)$

[4 pts] ii) Find β such that at $t = 0$ the contribution to $g(t)$ due to the pole at -5 is -5 times the contribution due the pole at -1.

$\beta =$ ____

Problem 2 Steady State Error (22 pts)

[6 pts] a) For the system at the bottom of the page, let $D(s) = 0$, $G_2(s) = 1$, $G_1(s) = \frac{k}{s+4}$, $H(s) = \frac{1}{s+1}$. That system can be redrawn in unity gain feedback form, as shown here. Determine the new open-loop transfer function $G(s)$ such that $\frac{C(s)}{R(s)}$ is the same in both cases.



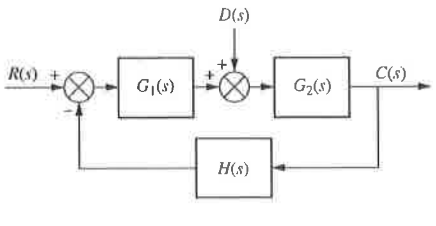
$G(s) =$ __________

[8 pts] b) For the system below, let $D(s) = 0$, $G_2(s) = 1$, $G_1(s) = \frac{k}{s+4}$, $H(s) = \frac{1}{s+1}$. Let $e(t) = r(t) - c(t)$. For $r(t) = u(t)$, a unit step, find the steady state expression for $e(t)$ for large t.

$e(t) =$ ______

[8 pts] c) For the system below, let $H(s) = \frac{s+5}{s+10}$, $G_1(s) = \frac{k(s+1)}{s}$, and $G_2(s) = 1$. For $d(t) = tu(t)$, a unit ramp, and $r(t) = 0$, find the steady state expression for $c(t)$ for large t.

$c(t) =$ ______



Problem 3. Routh-Hurwitz (14 pts)

Given system with closed loop transfer function (assuming unity feedback)

$$T(s) = \frac{k}{s^4 + 4s^3 + 6s^2 + 4s + 1 + k}$$

[10 pts] a. Using the Routh-Hurwitz table, find the range of k for which the closed loop system is stable.

$\_\_\_\_ < k < \_\_\_\_$

[4 pts] b. For the positive value of k found above, find the pair of closed loop poles on the imaginary axis. (Show work).

$s = \pm j\omega_o = \pm$ ____ j

Problem 4. Root Locus Plotting (27 pts)

Given open loop transfer function $G(s)$:

$$G(s) = \frac{(s+4)}{(s^2+2s+2)(s^2+4s+8)}$$

For the root locus $(1 + kG(s) = 0)$ with $k > 0$:

[1 pts] a) Determine the number of branches of the root locus = ___

[2 pts] b) Determine the locus of poles on the real axis ___________

[3 pts] c) Determine the angles for each asymptote: ___________

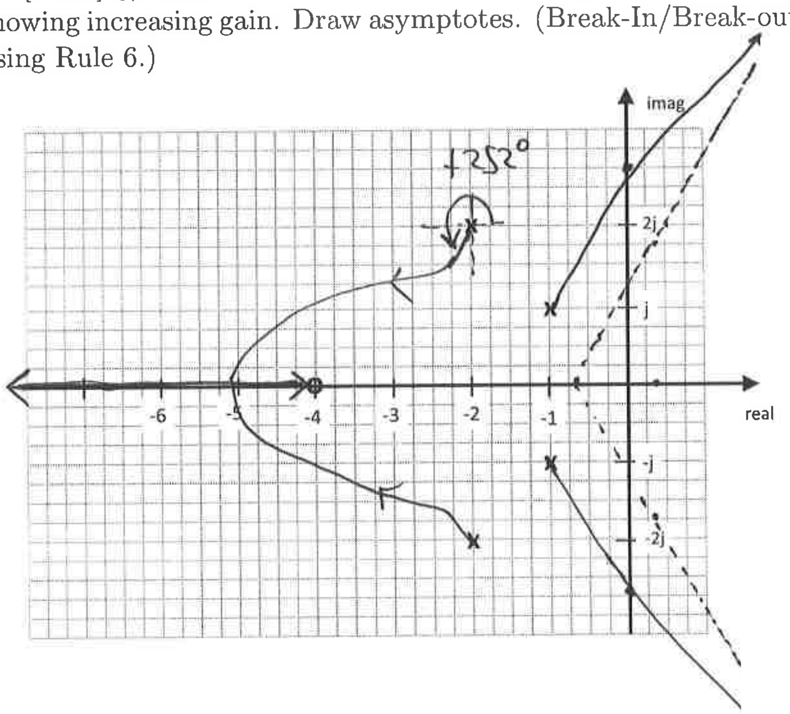
[4 pts] d) determine the real axis intercept for the asymptotes s =_____

[6 pts] e) Determine the angle of departure for the root locus for the pole at $s = -2 + 2j$ =_____

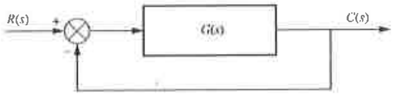
[6 pts] f) Estimate the value of k for which the closed loop system has a pole at $s \approx +2.5j$. ____

k = _____

[5 pts] g) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. Draw asymptotes. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)



Problem 5. Root Locus Compensation (15 pts)



Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{s+4}{(s+1)^2(s+5)^2}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a PD compensation of the form $G_1(s) = k\frac{s+z_c}{1}$. The closed loop system, using unity gain feedback and the PD controller, should have a pair of poles at $p = -2 \pm j$.

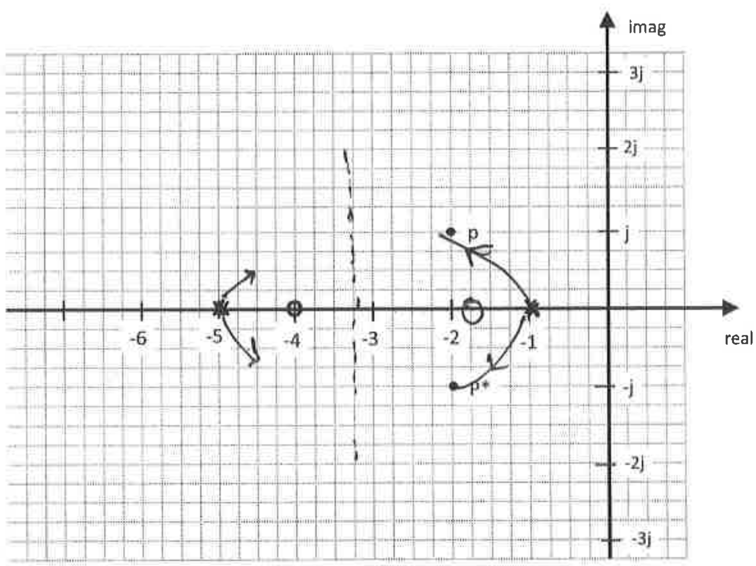
[4 pts] a. Show that for closed loop poles p, that the angle contribution contribution from $G_3(p)$ is $\approx -280°$.

[9 pts] b. Find z_c to within ± 0.1 such that p is approximately on the root locus, within $\pm$ 5 degrees. (Show work.)

	value	angle
z_c	1.8	+10 °

[2 pts] c. Briefly explain why or why not the closed loop system step response can be characterized by a dominant pole pair assumption, (with poles at $p = -2 \pm j$).

(Pole-Zero plot below for scratch work. It will not be graded).



page for scratch work