EECS C128/ ME C134
Midterm (v. 1.01)
Thurs. Mar. 9, 2017
0810-0930 am

Name: ____________________________  SID: ____________


- There are 5 problems worth 100 points total.

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td>22</td>
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<tr>
<td>3</td>
<td></td>
<td>14</td>
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<tr>
<td>4</td>
<td></td>
<td>27</td>
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<tr>
<td>5</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Tables for reference:

\[
\begin{align*}
\tan^{-1} \frac{1}{10} &= 5.7^\circ \\
\tan^{-1} \frac{1}{4} &= 14^\circ \\
\tan^{-1} \frac{1}{2} &= 26.6^\circ \\
\tan^{-1} \frac{1}{3} &= 33.7^\circ \\
\tan^{-1} \frac{1}{5} &= 45^\circ \\
\tan^{-1} \frac{1}{\sqrt{3}} &= 60^\circ \\
\sin 30^\circ &= \frac{1}{2} \\
\cos 30^\circ &= \frac{\sqrt{3}}{2} \\
\cos 45^\circ &= \frac{\sqrt{2}}{2} \\
\sin 45^\circ &= \frac{\sqrt{2}}{2}
\end{align*}
\]

<table>
<thead>
<tr>
<th>$20 \log_{10} 1$ = 0 dB</th>
<th>$20 \log_{10} 2$ = 6 dB</th>
<th>$\pi$ ≈ 3.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 \log_{10} \sqrt{2}$ = 3 dB</td>
<td>$20 \log_{10} \frac{1}{2}$ = -6 dB</td>
<td>$2 \pi$ ≈ 6.28</td>
</tr>
<tr>
<td>$20 \log_{10} 5$ = 20 dB - 6 dB = 14 dB</td>
<td>$20 \log_{10} \sqrt{10}$ = 10 dB</td>
<td>$\pi/2$ ≈ 1.57</td>
</tr>
<tr>
<td>$1/e$ ≈ 0.37</td>
<td>$\sqrt{10}$ ≈ 3.164</td>
<td>$\pi/4$ ≈ 0.79</td>
</tr>
<tr>
<td>$1/e^4$ ≈ 0.14</td>
<td>$\sqrt{2}$ ≈ 1.41</td>
<td>$\sqrt{3}$ ≈ 1.73</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{5}$ ≈ 2.24</td>
<td>$\sqrt{7}$ ≈ 2.65</td>
</tr>
<tr>
<td>$1/e^3$ ≈ 0.05</td>
<td>$1/\sqrt{2}$ ≈ 0.71</td>
<td>$1/\sqrt{3}$ ≈ 0.58</td>
</tr>
</tbody>
</table>
Problem 1 (22 pts)

Each part is independent.

[6 pts] a) Consider a single-input single-output system with transfer function:

\[
\frac{Y(s)}{U(s)} = \frac{2s^2 + 4s + 8}{s^2 + 6s + 10}
\]

input \(u(t)\) and output \(y(t)\). Choose coefficients such that the block diagram below has the same transfer function:

\[
b_0 = \_ \quad b_1 = \_ \quad b_2 = \_ \quad b_3 = \_ \quad b_4 = \_
\]

\[
a_1 = \_ \quad a_2 = \_ \quad a_3 = \_ \quad a_4 = \_
\]

[8 pts] b) Draw the equivalent electrical circuit for this mechanical system, with voltage corresponding to torque and current to rotational velocity. Input torque is \(T(t)\).

\[
\begin{array}{c}
\text{8 N-m/s/rad} \\
\text{5 kg-m²} \\
\text{1 N-m/s/rad} \\
\text{3 N-m/rad} \\
\end{array}
\]
c) Consider the system

\[ G(s) = \frac{s + \beta}{s(s + 1)(s + 5)} \cdot \frac{5}{\beta} \]

[4 pts] i) Find \( g(t) \) the inverse Laplace transform of \( G(s) \).

\[ g(t) = ( -e^{-t} + e^{-5t} + \ldots )u(t) \]

[4 pts] ii) Find \( \beta \) such that at \( t = 0 \) the contribution to \( g(t) \) due to the pole at \(-5\) is -5 times the contribution due the pole at -1.

\( \beta = \ldots \)
Problem 2 Steady State Error (22 pts)

[6 pts] a) For the system at the bottom of the page, let \( D(s) = 0 \), \( G_2(s) = 1 \), \( G_1(s) = \frac{k}{s+4} \), \( H(s) = \frac{1}{s+1} \). That system can be redrawn in unity gain feedback form, as shown here. Determine the new open-loop transfer function \( G(s) \) such that \( \frac{C'(s)}{R'(s)} \) is the same in both cases.

\[ G(s) = \] 

[8 pts] b) For the system below, let \( D(s) = 0 \), \( G_2(s) = 1 \), \( G_1(s) = \frac{k}{s+4} \), \( H(s) = \frac{1}{s+1} \). Let \( e(t) = r(t) - c(t) \). For \( r(t) = u(t) \), a unit step, find the steady state expression for \( e(t) \) for large \( t \).

\[ e(t) = \] 

[8 pts] c) For the system below, let \( H(s) = \frac{s+5}{s+10} \), \( G_1(s) = \frac{k(s+1)}{s} \), and \( G_2(s) = 1 \). For \( d(t) = tu(t) \), a unit ramp, and \( r(t) = 0 \), find the steady state expression for \( c(t) \) for large \( t \).

\[ e(t) = \]
Problem 3. Routh-Hurwitz (14 pts)

Given system with closed loop transfer function (assuming unity feedback)

\[ T(s) = \frac{k}{s^4 + 4s^3 + 6s^2 + 4s + 1 + k} \]

[10 pts] a. Using the Routh-Hurwitz table, find the range of \( k \) for which the closed loop system is stable.

\[ \quad < k < \quad \]

[4 pts] b. For the positive value of \( k \) found above, find the pair of closed loop poles on the imaginary axis. (Show work).

\[ s = \pm j\omega = \pm \quad \]

\[ \sqrt[3]{-1} \]
Problem 4. Root Locus Plotting (27 pts)
Given open loop transfer function \( G(s) \):

\[
G(s) = \frac{(s + 4)}{(s^2 + 2s + 2)(s^2 + 4s + 8)}
\]

For the root locus \((1 + kG(s) = 0) \) with \( k > 0 \):

[1 pts] a) Determine the number of branches of the root locus = __

[2 pts] b) Determine the locus of poles on the real axis __________

[3 pts] c) Determine the angles for each asymptote: __________

[4 pts] d) Determine the real axis intercept for the asymptotes \( s = \) __________

[6 pts] e) Determine the angle of departure for the root locus for the pole at \( s = -2 + 2j = \) __________

[6 pts] f) Estimate the value of \( k \) for which the closed loop system has a pole at \( s \approx +2.5j \). __________

\( k = \) __________

[5 pts] g) Sketch the root locus below using the information found above. Draw arrows on branches showing increasing gain. Draw asymptotes. (Break-In/Break-out points, if any, do not need to be calculated using Rule 6.)
Problem 5. Root Locus Compensation (15 pts)

Given open loop transfer function $G(s)$:

$$G(s) = G_1(s)G_3(s) = G_1(s)\frac{s + 4}{(s + 1)^2(s + 5)^2}$$

where $G_3(s)$ is the open-loop plant, and $G_1(s)$ is a PD compensation of the form $G_1(s) = k\frac{s + \frac{2}{k}}{s}$.

The closed loop system, using unity gain feedback and the PD controller, should have a pair of poles at $p = -2 \pm j$.

[4 pts] a. Show that for closed loop poles $p$, that the angle contribution contribution from $G_3(p)$ is $\approx -280^\circ$.

[9 pts] b. Find $z_c$ to within $\pm 0.1$ such that $p$ is approximately on the root locus, within $\pm 5$ degrees. (Show work.)

<table>
<thead>
<tr>
<th>$z_c$</th>
<th>value</th>
<th>angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/8</td>
<td>$\pi$</td>
<td>$\frac{\pi}{3}$</td>
</tr>
</tbody>
</table>

[2 pts] c. Briefly explain why or why not the closed loop system step response can be characterized by a dominant pole pair assumption, (with poles at $p = -2 \pm j$).

(Pole-Zero plot below for scratch work. It will not be graded.)