EE 40: Introduction to Microelectronic Circuits
Spring 2008: Midterm 1

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Total Time Allotted : 50 min
Total Points: 50

1. This is a closed book exam. However, you are allowed to bring one page (8.5 x 11) notes, with writing on both sides.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Show the steps used to arrive at your answer, where necessary. Partial credit will be given if you have the proper steps but an incorrect answer. A correct answer for a problem involving multiple steps where it is not clear how you arrived at the answer may not be given full credit.
4. Write your final answers into the boxes.
5. Remember to put down units.

Last (Family) Name:			
First Name:			
Student ID:		Lab Section:	
Signature:			

		Max	Score
1	(a)	4	
1	(b)	4	
2	(a)	3	
2	(b)	3	
2	(c)	3	
2	(d)	3	
2	(e)	3	
2	(f)	3	
3		10	
4		6	
5	(a)	3	
5	(b)	5	
Total		50	

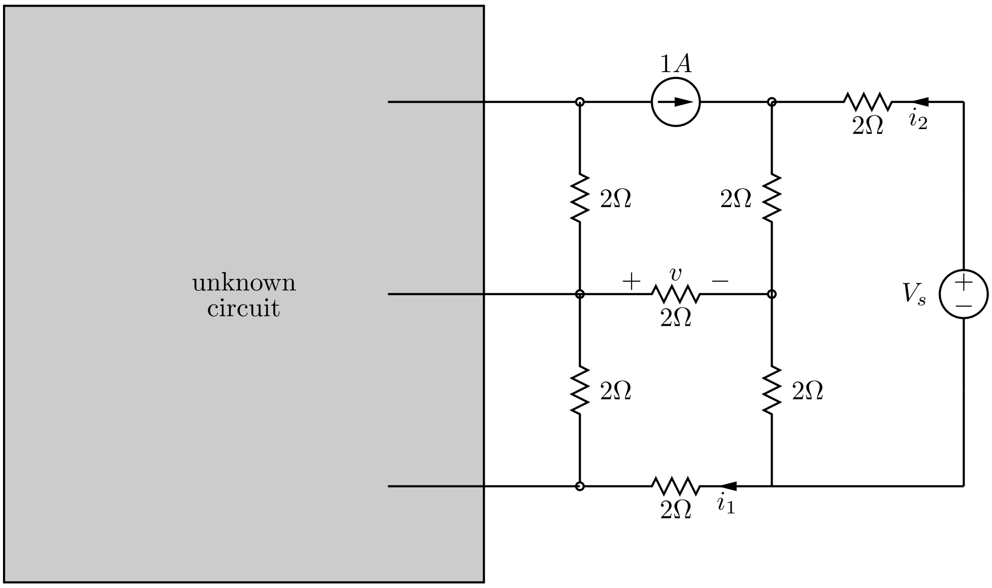


Figure 1: Circuit 1

1. In the circuit in Figure 1, each of the seven visible resistors has value 2Ω.
Unfortunately, the designers forgot to specify what is in the part of the circuit in the box on the left.
The voltage V_s is also unknown.
You are given that v equals $5V$.

   (a) Find i_1.
   Firstly, we recognize that the current through the 2Ω resistor in the middle is

$$i_m = \frac{v}{2\Omega} = \frac{5}{2}A$$

   Next, we consider a supernode around the unknown circuit and the two vertical aligned resistors next to it and write down the corresponding KCL equation:

$$i_1 = 1A + i_m = \frac{7}{2}A$$

$$\boxed{i_1 = \frac{7}{2}A}$$

   (b) Find i_2 in terms of V_s.
   Setting up the loop equation (KCL) on right side yields

$$V_s = i_2 2\Omega + (i_2 + 1A)2\Omega + (i_2 + 1A + i_m)2\Omega$$

   We solve for i_2

$$V_s - 2V - 7V = i_2 6\Omega$$
$$i_2 = \frac{V_s - 9V}{6\Omega}$$

$$\boxed{i_2 = \frac{V_s - 9V}{6\Omega}}$$

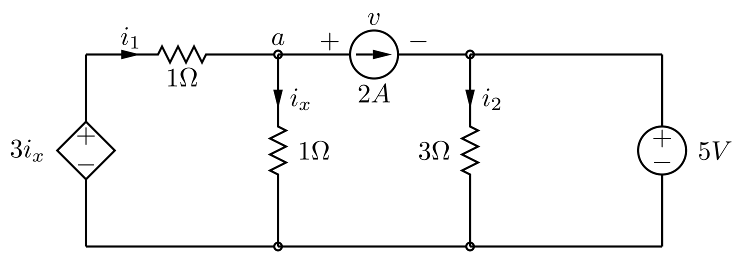


Figure 2: Circuit 2

2. Consider the circuit in Figure 2.

   (a) Use KCL at node a to find i_1 in terms of i_x.
   We write down the KCL equation.

$$i_1 = i_x + 2A \tag{1}$$

$$\boxed{i_1 = i_x + 2A}$$

   (b) Use KVL around the left loop to find i_x.
   Using KVL around the left loop yields

$$3i_x = i_1 + i_x \tag{2}$$

   Combining 1 and 2 gives us

$$3i_x = i_x + 2A + i_x$$

$$\boxed{i_x = 2A}$$

   (c) Use KVL around the right loop to find i_2.
   We can directly apply Ohm's law.

$$i_2 3\Omega = 5V$$

$$\boxed{i_2 = \frac{5}{3}A}$$

   (d) Find v. We use KVL around the middle loop

$$i_x 1\Omega = v + i_2 3\Omega$$

$$\boxed{v = -3V}$$

   (e) How much power is being delivered by the current source? *(Note: If power is delivered to the current source your answer will be a negative number)*
   Note that passive sign convention is used. Hence, you calculate the power with $p = v2A = (-3V)2A = -6W$
   A negative number means that power is delivered by the source.

   Power delivered by the current source:

   $6W$

(f) How much power is the independent voltage source delivering to the circuit?
Using KCL at the right upper node yields that there is a downward current of $\frac{1}{3}A$ through the voltage source. We calculate the power with the formula for passive sign convention

$$p = 5V\frac{1}{3}A = \frac{5}{3}W$$

The value is positive, hence the voltage source is absorbing power.

Power delivered by the independent voltage source: $-\frac{5}{3}W$

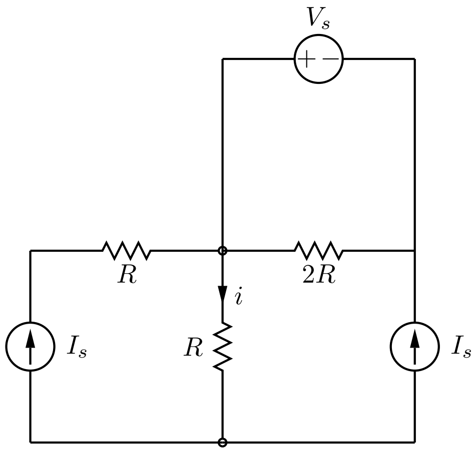


Figure 3: Circuit 3

3. In the circuit in Figure 3, use superposition to find the current i in terms of R, V_s and I_s.
(Note: You MUST use superposition to answer this question. Solutions using other techniques will not be accepted. Show the details of your work.)
Zeroing all sources with the exception of the current source on the right yields the circuit in Figure 4. All current from the current source is flowing through the center resistor. Hence,

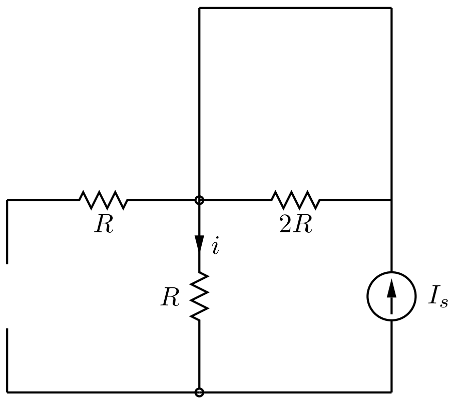


Figure 4: Circuit 7

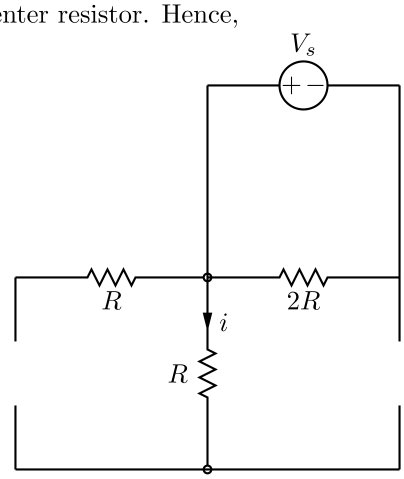


Figure 5: Circuit 8

$$i|_1 = I_s \tag{3}$$

Secondly, the two current sources are zeroed, calculating the impact of the voltage source on the current through the center resistor. The Circuit is depicted in Figure 5.
We see that no current is flowing through the center resistor.

$$i|_2 = 0 \tag{4}$$

Lastly, we consider the impact of the left current source and zero the right current source and the voltage source. We get the circuit in Figure 6. We directly see that all the current from the current source is flowing through the center resistor

$$i|_3 = I_s \tag{5}$$

Now we apply the superposition principle and calculate the total current through the center resistor from equations (3), (4), and (5).

$$i = i|_1 + i|_2 + i|_3 = 2I_s$$

$$\boxed{i = 2I_s}$$

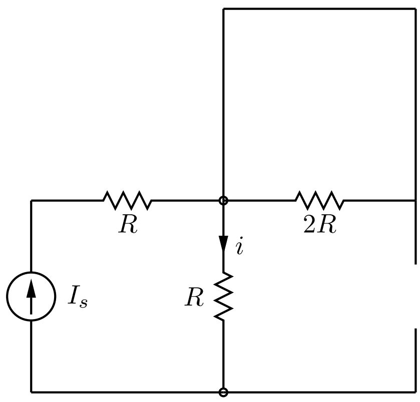


Figure 6: Circuit 9

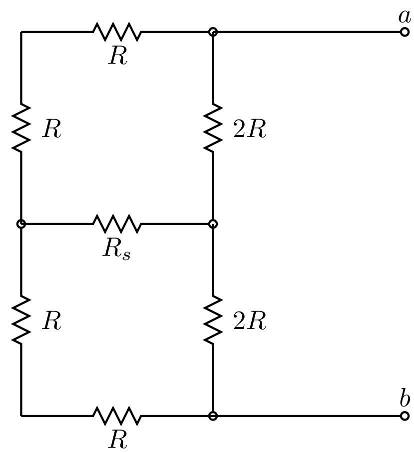


Figure 7: Circuit 4

4. Consider the resistive circuit in Figure 7. Find the equivalent resistance R_{ab} across the terminals a and b in terms of R and R_s.
By symmetry, we know that there is no current through resistor R_s. Hence, we can analyze the circuit as if this resistor was not there.

$$R_{ab} = (2R + 2R)||(R + R + R + R) = 2R$$

$$\boxed{R_{ab} = 2R}$$

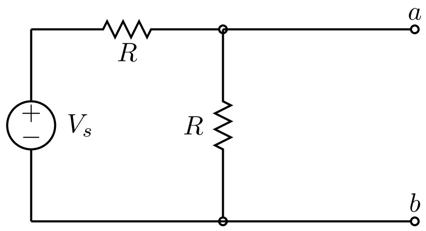


Figure 8: Circuit 5

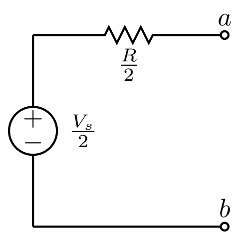
5. (a) Considering the circuit in Figure 8, find the Thévenin equivalent circuit across the terminals a and b.
We first zero the independent voltage source in order to find the Thévenin resistance. We get a circuit with two resistors with value R in parallel. Hence, we have

$$R_{th} = R||R = \frac{R}{2}$$

Next, we calculate the open circuit voltage. Here we know that the current through the terminals a and b is both $0A$ and we can use the voltage divider formula to get

$$v_{th} = V_s \frac{R}{R+R} = \frac{V_s}{2}$$

Thévenin Equivalent Circuit:



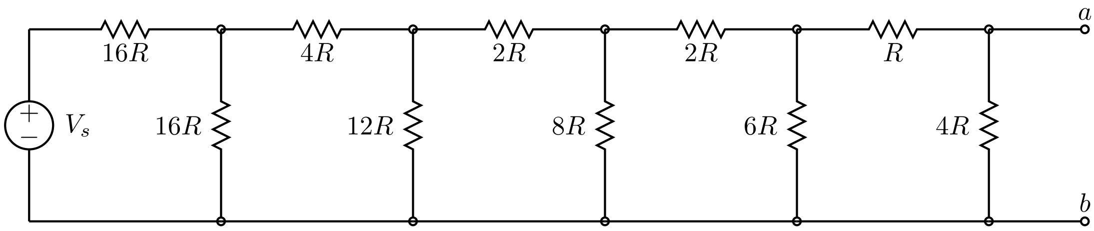


Figure 9: Circuit 6

(b) Using your answer from part (a), find the Thévenin equivalent circuit for the circuit in Figure 9 across the terminals a and b.
(Important: DO NOT USE BRUTE FORCE! Think about how you can use the answer of part (a) to make the calculation easy and then answer the problem.)
Explain your approach in a couple of lines.
The structure of the circuit in Figure 9 allows multiple application of the result of part (a): We can transform the voltage source and the two resistors on the left into a Thévenin equivalent circuit with $v_{th} = \frac{V_s}{2}$ and $R_{th} = 8R$.
As the next step, we combine R_{th} with the $4R$-resistor next to it which yields a Thévenin circuit with $R_{th} = 12R$.
Inspecting the resulting circuit, we realize that we have another instance of the circuit from (a). We apply this principle recursively through the entire circuit. Finally, we get

$$R_{th} = 2R$$

and

$$V_{th} = \frac{V_s}{32}$$

Thévenin Equivalent Circuit: