Midterm Exam # 2  
April 15, 2004  
Time Allowed: 80 minutes

Name:_______ SOLUTIONS_____, ______________________

Last                     First

Student ID #:______________, Signature:____________________

Discussion Section:________________________________________

This is a closed-book exam, except for use of two 8.5 x 11 inch sheet of your notes. Show all your work to receive full or partial credit. Write your answers clearly in the spaces provided.

<table>
<thead>
<tr>
<th>Problem #:</th>
<th>Points:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/10</td>
</tr>
<tr>
<td>2</td>
<td>/20</td>
</tr>
<tr>
<td>3</td>
<td>/20</td>
</tr>
<tr>
<td>Total</td>
<td>/50</td>
</tr>
</tbody>
</table>
1.

a) (5 points)
A silicon sample is uniformly doped with Boron to a concentration of $10^{16}$ atoms/cm$^3$. Determine the resistivity of the sample at room temperature.
Use electron mobility $\mu_n = 1000$ cm$^2$/V-s, hole mobility $\mu_p = 400$ cm$^2$/V-s, $Q = 1.6 \cdot 10^{-19}$ C and $n_i = 10^{10}$ at room temperature.

$$Na = 10^{16} \text{ cm}^{-3} \quad p = 10^{16} \text{ cm}^{-3} \quad > n_i$$

$p$ - type:

$$\rho = \frac{1}{q \mu_p} = \frac{1}{1.6 \cdot 10^{-19} \text{ C} \cdot 10^{14} \text{ cm}^{-3} \cdot 400 \text{ cm}^2/\text{V-s}} = \frac{1}{0.64} \text{ R cm}$$

$$= 1.56 \text{ R cm}$$

b) (5 points)

The same sample is then to be counter doped to a depth of 5 $\mu$m with Arsenic atoms to create a resistor technology with resistance of $100 \Omega/\square$.
Determine the required Arsenic doping density.

$$Rs = \frac{R'}{t}$$

$$R' = Rs \cdot t = 100 \text{ R} \cdot 5 \mu m = 0.05 \text{ R cm}$$

$p'$ - $p$ the sample becomes $n$ - type

$$p' = \frac{1}{q n \mu_n + q p \mu_p} = \frac{1}{q n \mu_n} = \frac{1}{q(N_d - Na) \mu_n}$$

$$Nd - Na = n = \frac{1}{q \mu_n p'} = \frac{1}{1.6 \cdot 10^{-19} \text{ C} \cdot 10^{14} \text{ cm}^{-3} \cdot 400 \text{ cm}^2/\text{V-s} \cdot 0.05 \text{ R cm}}$$

$$= 1.25 \cdot 10^{17} \text{ cm}^{-3}$$

$$Nd = n + Na = 1.25 \cdot 10^{17} \text{ cm}^{-3} + 10^{16} \text{ cm}^{-3}$$

$$= 1.35 \cdot 10^{17} \text{ cm}^{-3}$$
2. a) (10 points)

The diode in Figure 2(a) is ideal. The waveform \( V_S(t) \) is a balanced square wave with amplitude of 10 V and period 1 mS. Take \( L = 50 \mu H \) and \( R = 1 \Omega \).

The circuit operates in a periodic steady state. Sketch and carefully dimension one period of the \( i_L(t) \) waveform on the axes below. Make reasonable approximations.

\[
\tau = \frac{L}{R} = 0.05 \text{ms}
\]

\[
0.69 \tau = 0.0345 \text{ms} = 34.5 \mu \text{s}
\]
b) (10 points)

In the circuit of Figure 2(b), switch $S_1$ is initially closed and switch $S_2$ is initially open and the circuit is in equilibrium. Switch $S_1$ is then opened and switch $S_2$ is closed for a sufficiently long time so that the circuit can be considered to be in equilibrium. How much energy is dissipated in the 1 kΩ resistor during the transient?

**Hint:** Think in terms of net charge and energy flow. Detailed transient analysis is NOT needed.

$$\Delta W_c = \frac{1}{2} CV_2^2 - \frac{1}{2} CV_1^2$$
$$= \frac{1}{2} C \left( 100V^2 - 25V^2 \right)$$
$$= \frac{1}{2} \times 10^{-11} F \times 75V^2$$
$$= 3.75 \times 10^{-11} J$$

Total energy delivered by voltage source $V_2$:

$$W_{source} = \int V_2 i \, dt = V_2 \int i \, dt = V_2 \Delta Q$$
$$= V_2 \left( CV_2 - CV_1 \right)$$
$$= 5 \times 10^{-11} J$$

Energy dissipated in resistor:

$$W_R = W_S - \Delta W_c = 1.25 \times 10^{-11} J$$
Mosfet M1 in Figure 3 is modeled by 

\[ i_D = \frac{1}{2} k \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right) \]

in saturation with parameters listed in Figure 3.

a) (5 points)

Determine the required bias voltage \( V_G \) so that M1 is biased in saturation with \( V_{DS} = 2 \text{ V} \). Take \( V_S = 0 \)

\[ i_D = \frac{1}{2} k \left( V_{GS} - V_T \right)^2 \left( 1 + \lambda V_{DS} \right) \]

\[ \Rightarrow 120 \mu A = \frac{1}{2} \times 100 \mu A / \sqrt{2} \times 2 \times \left( V_{GS} - 0.5 \text{ V} \right)^2 \left( 1 + 0.1 V^{-1} \times 2 \text{ V} \right) \]

\[ \Rightarrow V_{GS} = 1.5 \text{ V} \]
b) (10 points)
Draw the small signal model for this circuit. Compute the parameters of this small signal model.

\[ g_m = \frac{2i_p}{V_{gs}} = k' \frac{W}{L} (V_{gs} - V_T) (1 + AV_{DS}) \]
\[ = 100 \text{mA/V}^2 \times 2 \times (1.5V - 1V) (1 + 0.1 \times 1V^{-1} - 2V) \]
\[ = 2.4 \times 10^{-4} \leq \]

\[ g_0 = \frac{2i_p}{V_{DS}} = \frac{1}{2} k' \frac{W}{L} (V_{gs} - V_T)^2 \cdot x \]
\[ = \frac{1}{2} \times 100 \text{mA/V}^2 \times 2 \times (1.5V - 1V)^2 \times 0.1 \times 1V^{-1} \]
\[ = 10^{-5} \leq \]

c) (5 points)
Determine the small signal gain

\[ A_v = \frac{v_o}{v_s} \]

\[ v_o = -g_m v_{gs} \cdot r_o \]
\[ = -g_m v_s \cdot \frac{1}{g_o} \]

\[ A_v = \frac{v_o}{v_s} = - \frac{g_m}{g_o} = - \frac{2.4 \times 10^{-4} \times 10^{-3}}{10^{-5}} = -24 \]