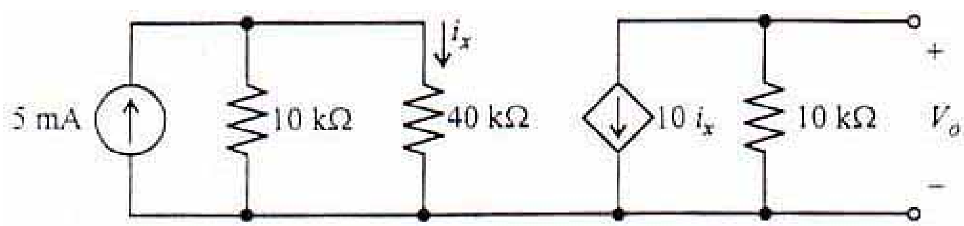
EE40 Midterm 2 Solutions
Spring 2000

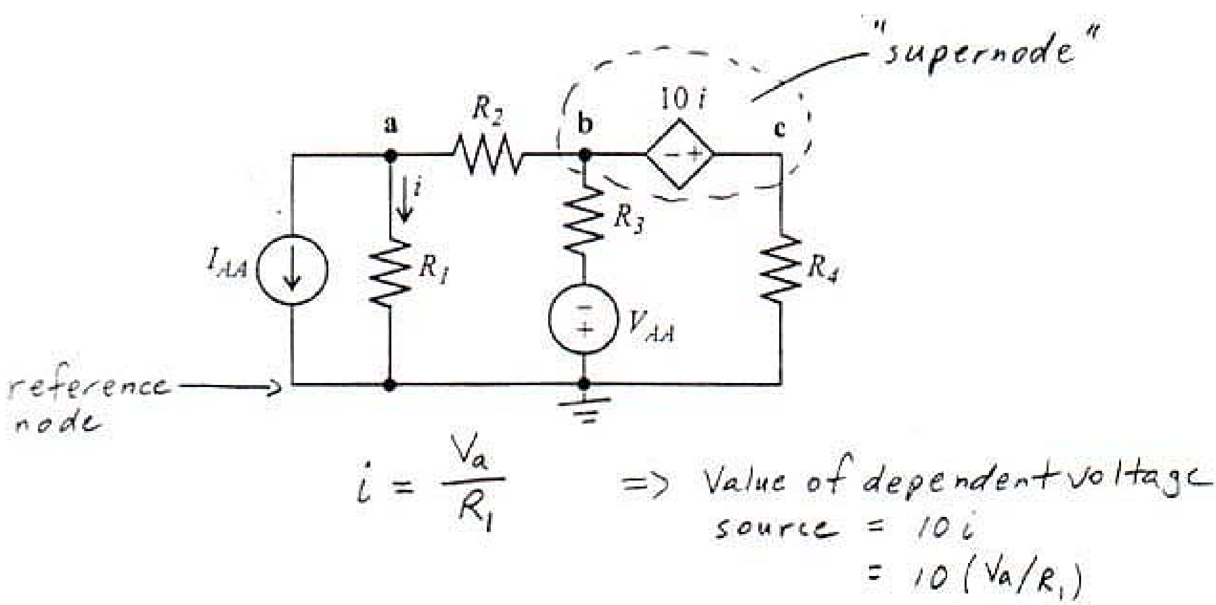
Problem #1: Circuits with Dependent Sources [20 points]

a) Find V_0. **[4 pts]**



Current divider formula: i_x = (10 kohm/(10 kohm + 40 kohm))*5 mA = 1 mA
V_0 = ($-10i_x$)(10 kohm) = (-10*1 mA)(10 kohm) = -100 V

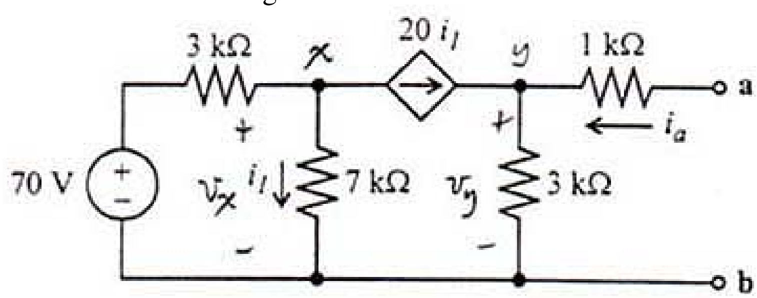
b) In the circuit below, the independent source values and resistance are known. Use the **nodal analysis technique** to write **3 equations sufficient to solve for Va, Vb, and Vc**. To receive credit, you must write your answer in the box below.
[6 pts] DO NOT SOLVE THE EQUATIONS!



Write the nodal equations here: Note that the only unknowns in these equations are Va, Vb, Vc

node a: $I_{AA} + V_a/R_1 + (V_a-V_b)/R_2 = 0$
supernode: $(V_b-V_a)/R_2 + (V_b+V_{AA})/R_3 + V_c/R_4 = 0$
relationship due to dependent source: $V_c-V_b = 10V_a/R_1$

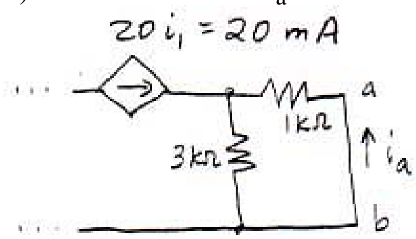
c) Consider the following circuit:



i) Find the voltage V_{ab} **[5 pts]**

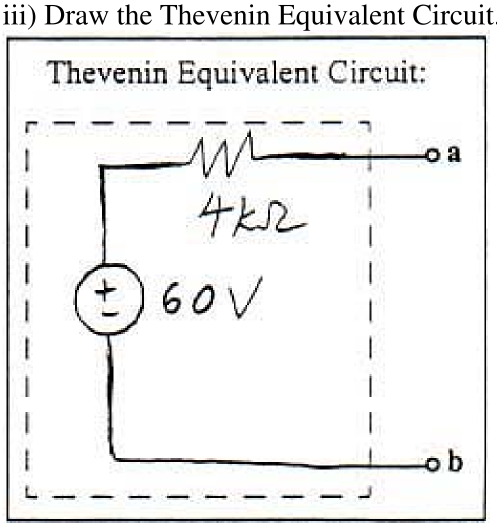
$i_1 = V_x/7$ kohm
Applying KCL to node x: $70-V_x/3$ kohm $= i_1 + 20i_1 = 21i_1 = 21V_x/7$ kohm
$70-V_x = 9V_x \Rightarrow V_x = 7$
$i_1 = 7$ V/7 kohm = 1 mA; $i_a = 0 \Rightarrow V_{ab} = V_y = 20i_1(3$ kohm$) = 60$V

ii) What is the current i_a when the terminals **a** and **b** are shorted together? **[3 pts]**



Current divider formula:
$i_a = (3$ kohm$/(3$ kohm $+ 1$ kohm$))(-20$ mA$) = -15$ mA

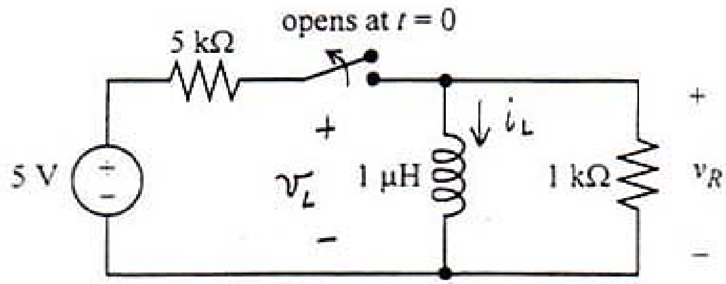
iii) Draw the Thevenin Equivalent Circuit. **[2 pts]**



$V_{th} = V_{oc} = V_{ab}$ from part (i)
Rth $= -V_{oc}/I_{sc} = V_{ab}$ from part (i)/i_a from part (ii) = 60 V/-15 mA = 4 kohm

Problem #2: Transient Response [30 points]

a) In the circuit below, the switch has been in the closed position for a long time.



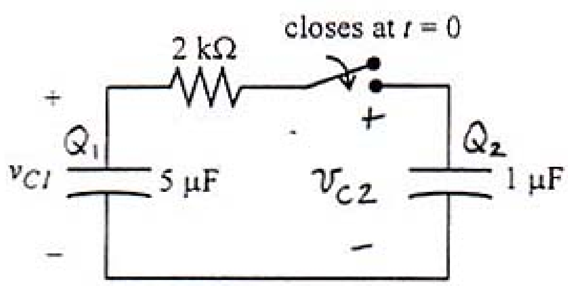
i) Find the value of V_R just after the switch opens (t = 0+). **[3 pts]**

Before the switch is opened, the voltage across the inductor is zero (V_L = Ldi/dt = 0), i.e. the 1 kohm resistor is shorted out by the inductor. The current flowing through the inductor $i_L(0^-)$ = 5 V/5 kohm = 1 mA.
$i_L(0^+) = i_L(0^-)$ = 1 mA, since inductor current cannot change instantaneously.
$V_R(0^+) = -i_L(0^+)$(1 kohm) = (-1 mA)(1 kohm) = -1 V

ii) How much energy is dissipated in the 1 kohm resistor after the switch is opened? **[2 pts]**

All of the energy which was stored in the inductor at t = 0 is dissipated in the 1 kohm resistor after the switch is closed.
$E = 0.5L[i(0)]^2 = 0.5(10^{-6}\text{ H})(10^{-3}\text{ A})^2 = 0.5*10^{-12}$ J = 0.5 pJ

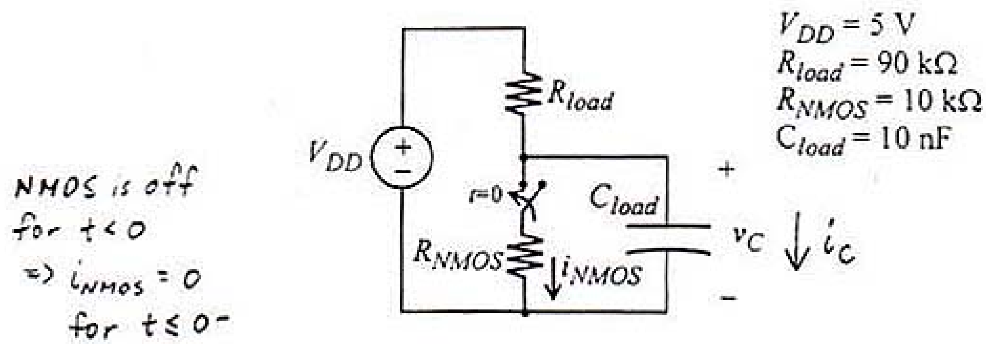
b) In the circuit below, the 5 microF capacitor is initially charged to 5 V ($V_{C1}(0^-)$ = 5 V). (The 1 microF capacitor is initially uncharged.) The switch is then closed at time t = 0. What is the final value of V_{C1}? **[5 pts]**



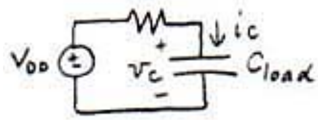
Conservation of charge:
$Q_{1final} + Q_{2final} = Q_{1initial}$
(5 microF)($V_{C1final}$) + (1 microF)($V_{C2final}$) = (5 microF)($V_{C1initial}$)

In the final state (t = 0), the voltages across the capacitors are equal, i.e. $V_{C1final} = V_{C2final}$
(6 microF)($V_{C1final}$) = (5 microF)(5 V)
$V_{C1final}$ = (5/6)(5 V) = 4.17 V ~ 4.2 V

c) The following is a circuit model for an NMOS inverter, in which the transistor is turned on at time t = 0:



i) What is the value of V_C at t = 0-? **[3 pts]**



In steady state (before the switch closes), $i_c = 0$; $V_c = V_{DD} = 5V$

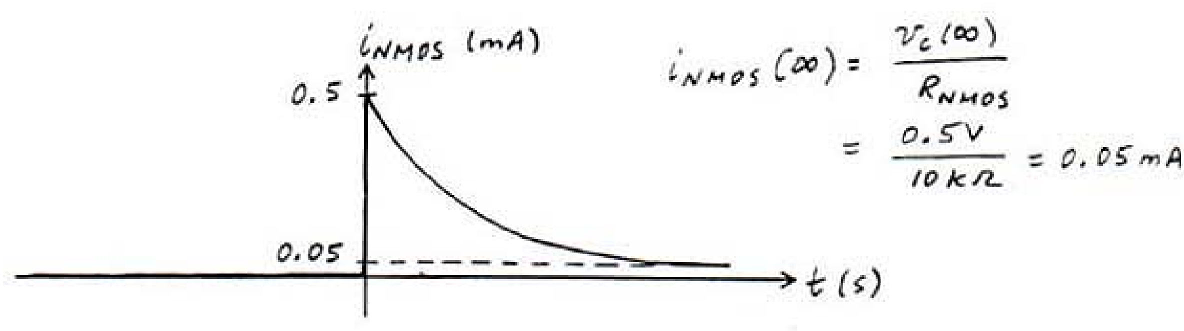
ii) What is the value of i_{NMOS} at t = 0+ **[3 pts]**

$V_c(0+) = Vc(0-)$, since capacitor voltage cannot change instantaneously.
$i_{NMOS}(0+) = V_c(0+)/R_{NMOS} = 5$ V/10 kohm = 0.5 mA

iii) What is the final value of V_c? **[3 pts]**

In steady state, $i_c = 0$. Therefore, we have a simple voltage divider:
$V_c(\text{inifinity}) = [R_{NMOS}/(R_{NMOS} + R_{load}]*V_{DD}$
= (10/(10+90))(5 V) = 0.5 V

iv) Neatly sketch the graph of i_{NMOS} for all t, labeling the axes. **[5 pts]**



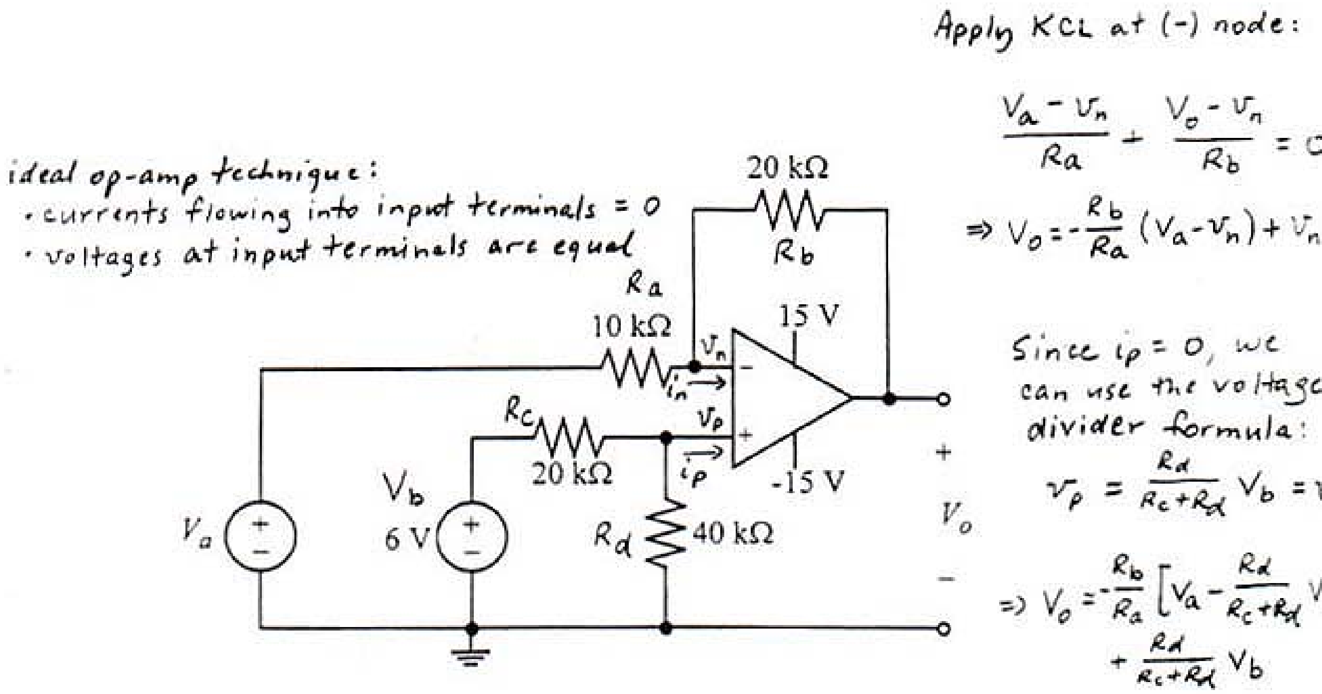
v) Write an equation for i_{NMOS} as a function of time, for t>0. **[6 pts]**

$i_{NMOS} = i_{NMOSfinal} + [i_{NMOS}(0+) - i_{NMOSfinal}]e^{-(t/ReqC)}$
R_{eq} is equivalent resistance seen by the capacitor: $R_{eq} = R_{load} \parallel R_{NMOS}$
R_{eq} = (90)(10)/(90+10) = 9 kohm; $R_{eq}C$ = (9 kohm)(10 nF) = $9*10^{-5}$ s
Equation: $0.05 + 0.45e^{-(t/9*10^{\wedge}-5)}$ mA

Problem #3: Op-Amp Circuits [25 points]

Assume the op-amps in this problem are ideal.

a) Consider the following circuit:



i) Find an expression for V_0 as a function of V_a. **[6 pts]**

This is a difference amplifier circuit (which you've studied in the lab) with $R_a/R_b = R_c/R_d = 1/2$
$V_0 = (R_b/R_a(6-V_a) = 2(6-V_a) = 12 - 2V_a$

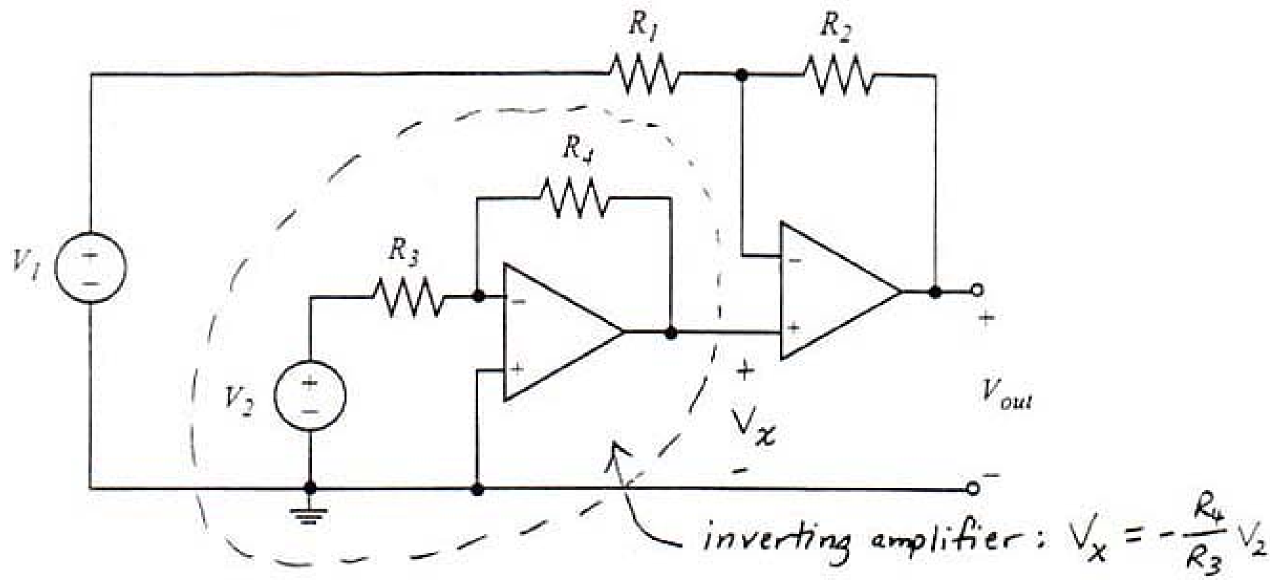
ii) Find V_0 for $V_a = 2$ V. **[3 pts]**

$V_0 = 12 - 2(2) = 8$ V

iii) For what values of V_a will the op-amp be saturated? **[6 pts]**

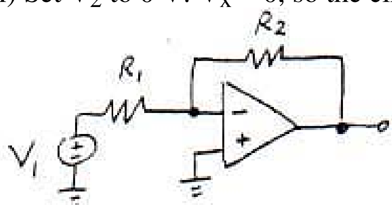
$V_0 = 12 - 2V_a \Rightarrow V_a = (12 - V_0)/2$
V_0 saturates at 15 V: $V_a <= (12-15)/2 = -3/2$ V
V_0 saturates at -15 V: $V_a >= (12 - (-15))/2 = 27/2$ V
Values of V_a for which the op-amp will be saturated: $V_a <= -1.5$ V; $V_a >= 13.5$ V

b) In the following circuit, the op=amps are operating linearly.



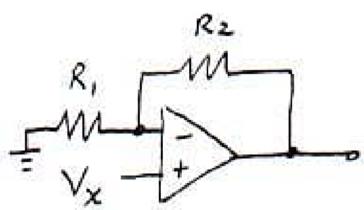
Find V_{out} in terms of V_1, V_2, R_1, R_2, R_3, R_4 **[10 pts]** (Hint: The superposition method might be helpful here.)

Find the individual contributions of each voltage source:
i) Set V_2 to 0 V: $V_x = 0$, so the circuit simplifies to a simple inverting amplifier



$$V'_{out} = -\frac{R_2}{R_1} V_1$$

ii) Set V_1 to 0 V: circuit simplifies to simple non-inverting amplifier

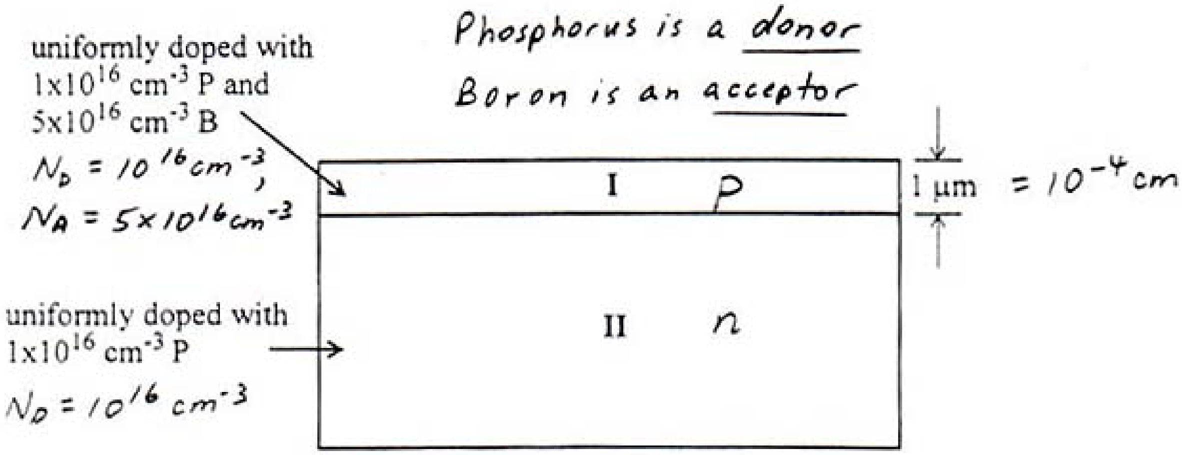


$$V''_{out} = \left(1+\frac{R_2}{R_1}\right)V_x = \left(1+\frac{R_2}{R_1}\right)\left(-\frac{R_4}{R_3}V_2\right)$$

Add the contributions of each source together:
$V_{out} = V_{out}' + V_{out}'' = -(R_2/R_1)(V_1) - (R_4/R_3)(1+(R_2/R_1))(V_2)$

Problem #4: Semiconductor properties; p-n diodes [25 points]

a) Consider a silicon sample maintained at 300K under equilibrium conditions, uniformly doped with $1*10^{16}$ cm^{-3} phosphorus atoms. The surface region of the sample is additionally doped uniformly with $5*10^{16}$ cm^{-3} boron atoms, to a depth of 1 microm, as shown in the figure below.



Schematic cross-sectional view of silicon sample

i) In the figure above, indicate the type of regions (I and II) by labelling them as "n" or "p" type. **[2 pts]**

ii) What are the electron and hole concentrations in Region I? **[5 pts]**

$N_A > N_D$, and $N_A >> n_i$ so $p = N_A - N_D = 5*10^{16} - 1*10^{16} = 4*10^{16}$
$pn = n_i^2 \Rightarrow n = n_i^2/p = (1.45*10^{16})2/4*10^{16} = 5256$
$n = 5256$ cm^{-3}; $p = 4*10^{16}$ cm^{-3}

iii) What is the sheet resistance of Region I? **[5 pts]**

rho = $1/(qmu_n n+qmu_p p) \sim 1/qu_p p$
>From plot on Page 2, $mu_p \sim 350$ cm^2/Vs for $N_A + N_D = 6*10^{16}$ cm^{-3}
R_s = rho/t = $1/qmu_p pt = [(1.602*10^{-19})(350)(4*10^{16})(10^{-4})]^{-1} = 4458$ ohm/square

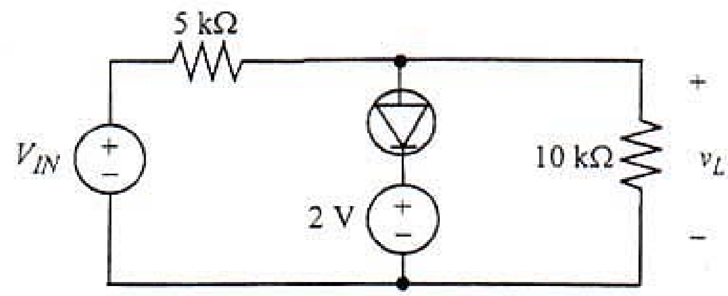
iv) Suppose any voltage between 0 V and 5 V can be applied to Region I. What fixed voltage ("bias") would you apply to Region II, to guarantee that no current would ever flow between Region I and Region II? Briefly explain your answer. **[3 pts]**

To prevent current from flowing, we need to ensure that the p-n junction will never be forward biased. Thus, the n-type region must be biased at 5 V or higher.

b) If a diode is operated only within a small range of forward-bias voltages, its behavior can be accurately modeled by a resistor, whose value is dependent on the bias voltage. Derive an expression for the diode "small-signal" resistance:
$R_{diode} = (dI/dV)^{-1}$ in terms of the saturation current Is, the bias voltage V, and the absolute temperature T. **[5 pts]**

$I = I_s(e^{qV/kT} - 1)$
$dI/dV = I_s(q/kT)e^{qv/kT}$
$R_{diode} = (kT/qI_s)e^{-qV/kT}$

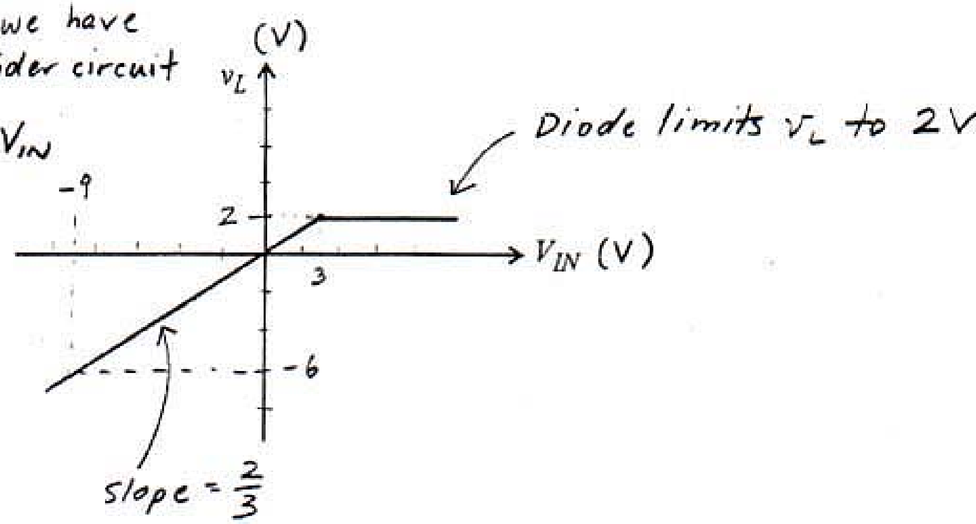
c) Plot V_L vs. V_{IN} for -10 V < V_{IN} < 10 V on the axes provided, for the circuit below. Note that the diode is a perfect rectifier. Label the axes. **[5 pts]**



When the diode is off, we have a simple voltage-divider circuit

$$v_L = \frac{10}{5+10} V_{IN} = \frac{2}{3} V_{IN}$$

Diode turns on when v_L reaches 2V, i.e. when $\frac{2}{3} V_{IN} = 2V$, or $V_{IN} = 3V$



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