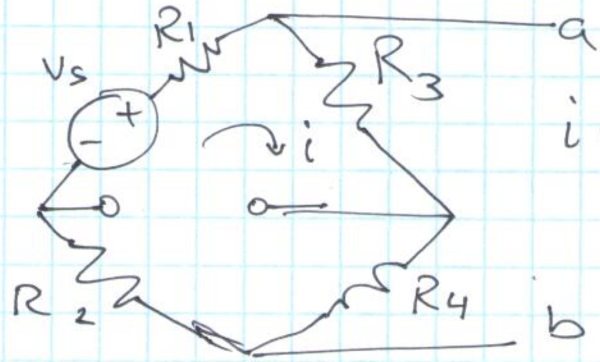
EE40 MT2.

1. $R_{th} = (R_1+R_2) \| (R_3+R_4) = \dfrac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$ — 10pts.

V_{th}: Need to use superposition thm. to make it easier to solve.
though Mesh and Nodal Analysis received credit if solved properly. (10 pts).

Examine effect of V_s only:



$$i = \frac{V_s}{R_1+R_2+R_3+R_4}$$

$$V_{ab}\big|_{V_s} = i(R_3+R_4).$$
$$= \frac{V_s(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

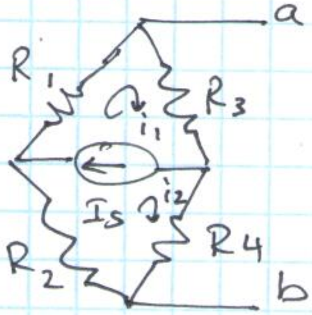
$$V_{ab} = V_{ab}\big|_{V_s} + \underbrace{V_{ab}\big|_{I_s}}.$$

↳ cont.d on next page.

points breakup:

R_{th}:
method: upto 5 pts
execution: upto 3 pts.
correct expression: 2 pts

V_{th}:
method: upto 6 pts
execution: upto 3 pts.
correct expr: 1 pts.

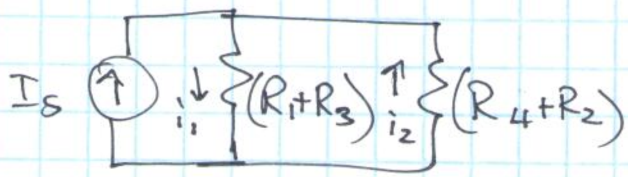


$I_S = i_1 - i_2$ —— ①.

$V_{ab}|_{I_s} = i_1 R_3 + i_2 R_4$ —— ②.

Apply KVL:

$$-i_2 R_2 - i_1 R_1 - i_1 R_3 - i_2 R_4 = 0.$$



$$i_1 (R_1+R_3) = -i_2 (R_2+R_4).$$

$$
\begin{aligned}
I_S (R_1+R_3) &= (R_1+R_3)\, i_1 - (R_1+R_3)\, i_2 \\
-\big(\quad 0 &= (R_1+R_3)\, i_2 + (R_2+R_4)\, i_2 \big) \\
\hline
I_S (R_1+R_3) &= -(R_1+R_2+R_3+R_4)\, i_2 .
\end{aligned}
$$

$$i_2 = = \frac{-I_S (R_1+R_3)}{R_1+R_2+R_3+R_4}$$

$$i_1 = i_2 + I_S = \frac{-I_S (R_1+R_3) + (R_1+R_2+R_3+R_4) I_S}{R_1+R_2+R_3+R_4}$$

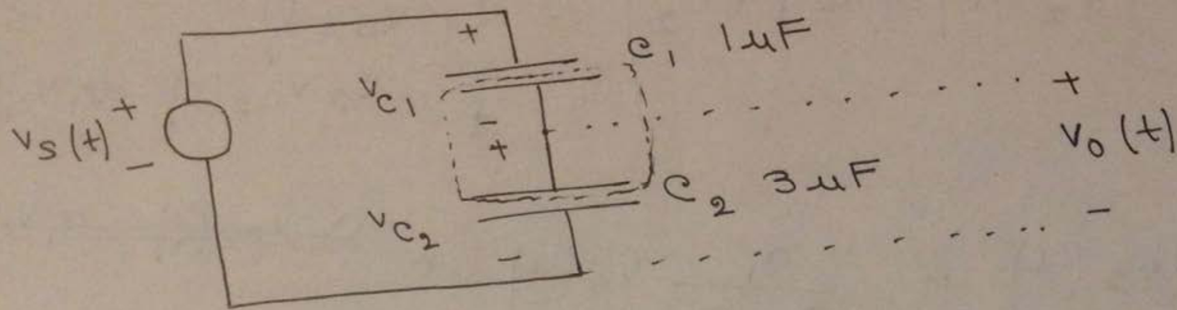
$$\Rightarrow i_1 = \frac{(R_2+R_4) I_S}{R_1+R_2+R_3+R_4}$$

$$\Rightarrow V_{ab}|_{I_s} = \frac{R_3 (R_2+R_4) I_S}{\Sigma R} - \frac{I_S (R_1+R_3) R_4}{\Sigma R}$$

$$= \frac{I_S}{\Sigma R} \left(R_2 R_3 + \cancel{R_3 R_4} - R_1 R_4 - \cancel{R_3 R_4} \right)$$

$$V_{ab}|_{I_s} = \frac{I_S (R_2 R_3 - R_1 R_4)}{R_1+R_2+R_3+R_4}$$

Soalution # 2



$v_{C_1}(t) + v_{C_2}(t) = v_S(t)$

At $t = 0$, $v_{C_1}(0) = -1\text{ V}$ and so $v_{C_2}(0) = 1\text{ V}$

because $v_S(0) = 0$

$v_{C_1}(t) + v_{C_2}(t) = v_S(t)$

$\cancel{C_1 v_{C_1}(t) = C_2 v_{C_2}(t)}$

Charge inside the dotted area is fixed

$\therefore$ $\cancel{\frac{C_2}{C_1}}$

$$-C_1 v_{C_1}(t) + C_2 v_{C_2}(t) = -C_1 v_{C_1}(0) + C_2 v_{C_2}(0)$$

$$\Rightarrow -C_1\left(v_S(t) - v_{C_2}(t)\right) + C_2 v_{C_2}(t) = C_2 v_{C_2}(0) - C_1 v_{C_1}(0)$$

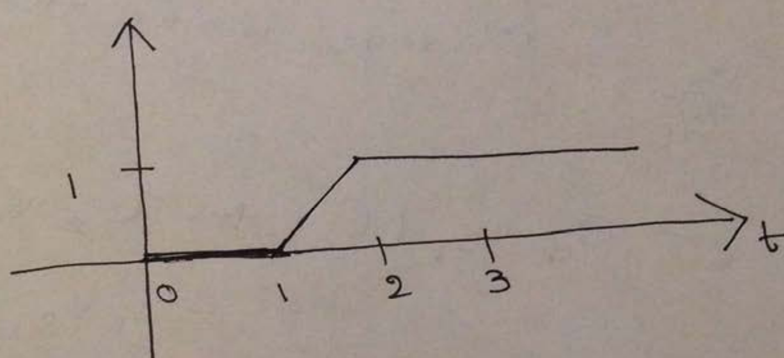
$$\Rightarrow \quad v_{C_2}(t)\,(C_1 + C_2) = C_1\, v_S(t) + C_2\, v_{C_2}(0) - C_1\, v_{C_1}(0)$$

$$\therefore \Rightarrow v_{C_2}(t) = \frac{C_1}{C_1 + C_2}\, v_S(t) + \frac{C_2\, v_{C_2}(0) - C_1\, v_{C_1}(0)}{C_1 + C_2} = v_{out}(t)$$

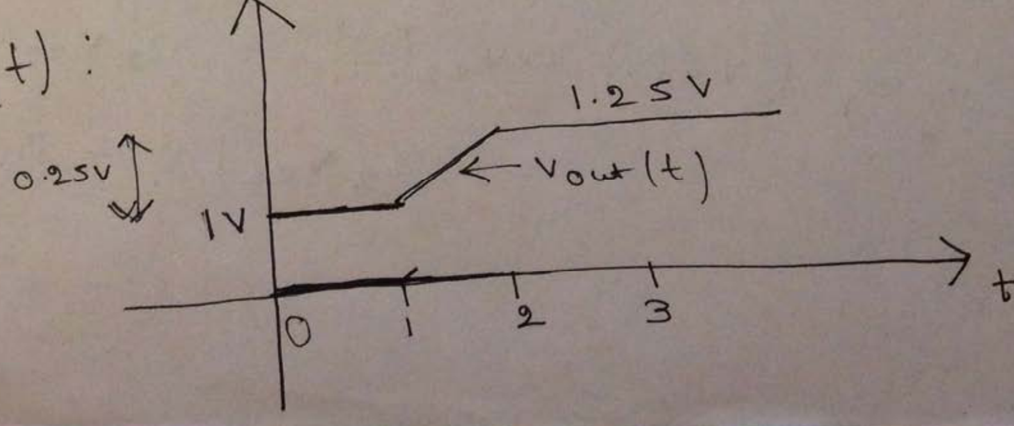
$$\therefore V_{out}(t) = v_{C_2}(t) = \frac{C_1}{C_1 + C_2}\, v_S(t) + \frac{C_2 + C_1}{C_1 + C_2}$$

$$= 1 + \frac{1}{4}\, v_S(t)$$

$\therefore$ For $v_S(t)$:



$V_{out}(t)$:



Rubric

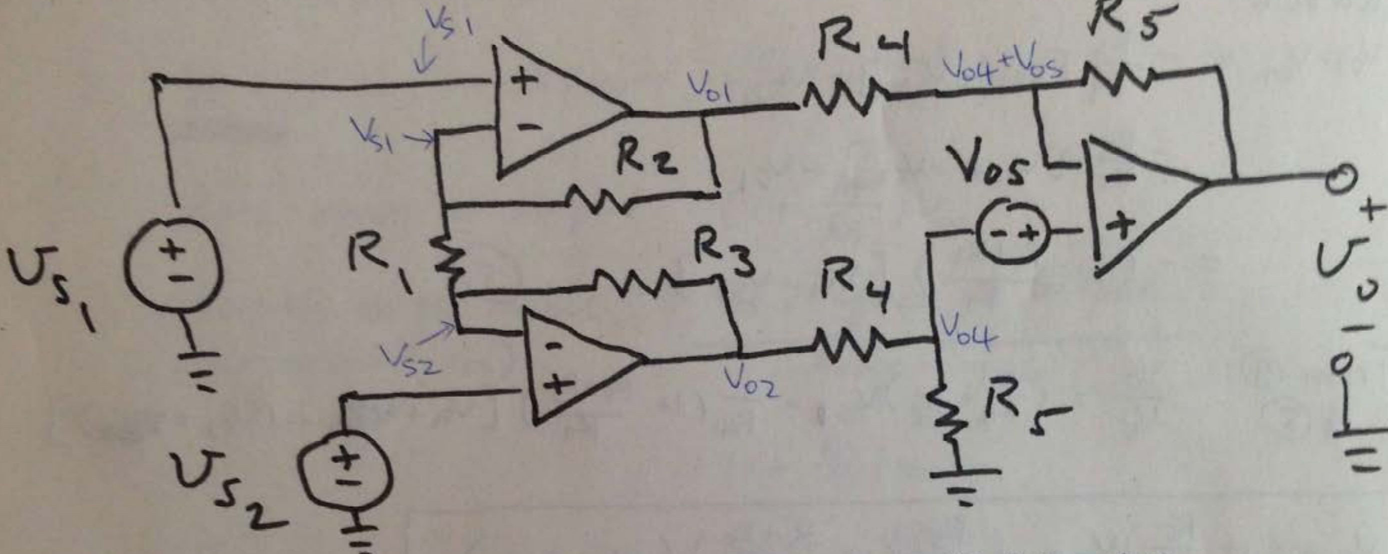
+5 for initial value ($V_o(0) = 1V$
and it stays flat till
$t = 1s$)

+5 for V_{out} going up
from $t = 1$ to $t = 2s$ on a
straight line

+5 for final value ($V_o(t=2)$
$= 1.25V$
~~$= V_o(0$~~
and it stays flat after that)

+5 for working

3) [30 points total] In the circuit below, each of the op-amps is ideal. Voltage source $\mathbf{V_{os}}$ is a constant voltage used to model a non-ideality associated with the op-amp on the right-hand side of the figure.



a) [20 points] Determine an expression for $\mathbf{v_o}$ in terms of the independent sources and indicated circuits parameters.

① $\frac{V_{o1}-V_{s1}}{R_2} = \frac{V_{s1}-V_{s2}}{R_1} = \frac{V_{s2}-V_{o2}}{R_3}$

$\Rightarrow V_{o1} = \frac{R_2}{R_1}[V_{s1}-V_{s2}] + V_{s1}$ - - - ①a

$V_{o2} = -\frac{R_3}{R_1}[V_{s1}-V_{s2}] + V_{s2}$ - - - ①b

② $\frac{V_o-(V_{o4}+V_{os})}{R_5} = \frac{V_{o4}+V_{os}-V_{o1}}{R_4}$

③ $\frac{V_{o4}-V_{o2}}{R_4} + \frac{V_{o4}}{R_5} = 0$

③ $\Rightarrow V_{o4}\left(\frac{1}{R_4}+\frac{1}{R_5}\right) = \frac{V_{o2}}{R_4}$

$V_{o4} = \frac{1}{\frac{1}{R_4}+\frac{1}{R_5}} \frac{V_{o2}}{R_4}$ ... ③'

② $\Rightarrow \frac{V_o}{R_5} = (V_{o4}+V_{os})\left(\frac{1}{R_4}+\frac{1}{R_5}\right) - \frac{V_{o1}}{R_4}$

$\overset{③'}{=} \left(\frac{1}{\frac{1}{R_4}+\frac{1}{R_5}} \frac{V_{o2}}{R_4} + V_{os}\right)\left(\frac{1}{R_4}+\frac{1}{R_5}\right) - \frac{V_{o1}}{R_4}$

$$\frac{V_o}{R_5} = \frac{V_{o2}}{R_4} + V_{os}\left(\frac{1}{R_4}+\frac{1}{R_5}\right) - \frac{V_{o1}}{R_4}$$

$$= \left(\frac{1}{R_4}+\frac{1}{R_5}\right)V_{os} + \frac{1}{R_4}\left(V_{o2}-V_{o1}\right) \quad - \cdots \text{(4)}$$

From (1)a & (1)b

$$V_{o2}-V_{o1} = -\frac{R_3}{R_1}\left[V_{s1}-V_{s2}\right] + V_{s2}$$

$$-\frac{R_2}{R_1}\left[V_{s1}-V_{s2}\right] - V_{s1}$$

$$= -\left(1+\frac{R_2+R_3}{R_1}\right)\left[V_{s1}-V_{s2}\right] \quad \cdots \text{(5)}$$

From (4) & (5):

$$\frac{V_o}{V_5} = \left(\frac{1}{R_4}+\frac{1}{R_5}\right)V_{os} - \frac{1}{R_4}\left(1+\frac{R_2+R_3}{R_1}\right)\left[V_{s1}+\cancel{\quad} - \left(V_{s2}+\cancel{\quad}\right)\right]$$

$$\boxed{V_o = \underbrace{\left(1+\frac{R_5}{R_4}\right)V_{os}}_{\text{Output DC shift due to } V_{os}} - \underbrace{\frac{R_5}{R_4}\left(1+\frac{R_2+R_3}{R_1}\right)}_{\text{Amp's gain}}\underbrace{\left(V_{s1}-V_{s2}\right)}_{\text{differential input.}}}$$

b) [10 points] For this circuit, if you have some freedom in choosing resistor values, what choice(s) should you make to minimize the effect of non-zero $\mathbf{V_{os}}$ relative to the signal inputs $\mathbf{v_{s1}}$ and $\mathbf{v_{s2}}$?

gain of the amplifier: $\frac{R_5}{R_4}\left(1+\frac{R_2+R_3}{R_1}\right)$

V_{os}'s effect at output: $\left(1+\frac{R_5}{R_4}\right)V_{os}$

refer V_{os} to the input : $\dfrac{V_{os}\text{'s effect at output}}{\text{Amp gain}}$
(input referred V_{os})

$$= \frac{\left(1+\frac{R_5}{R_4}\right)V_{os}}{\frac{R_5}{R_4}\left(1+\frac{R_2+R_3}{R_1}\right)}$$

$$= \frac{\left(1+\frac{R_4}{R_5}\right)V_{os}}{1+\frac{R_2+R_3}{R_1}}$$

want to minimize this input referred V_{os}, so we want to increase the gain of the amplifier, meaning increasing $\frac{R_5}{R_4}\left(1+\frac{R_2+R_3}{R_1}\right)$

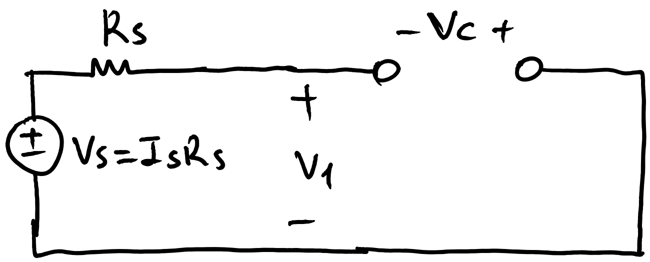
so increasing $\frac{R_2+R_3}{R_1}$ and/or ~~de~~creasing $\frac{R_4}{R_5}$

effective

not that effective

Q4

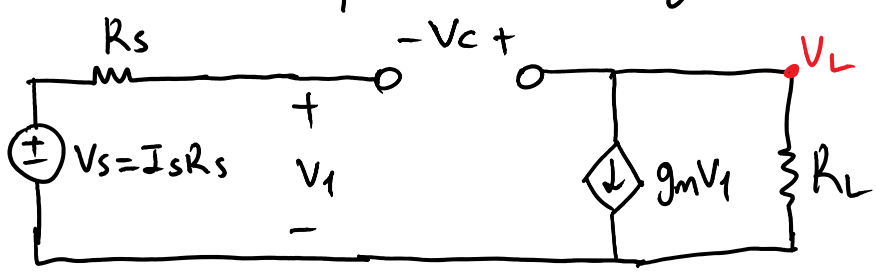
at $t=0^-$ capacitor is at steady state → open circuit



switch is closed

KVL: $V_c + I_s R_s = 0 \rightarrow \boxed{V_c(0^-) = -I_s R_s = V_i}$

at $t=0^+$ switch opens. At steady state ($t\to\infty$), capacitor becomes an open circuit again.



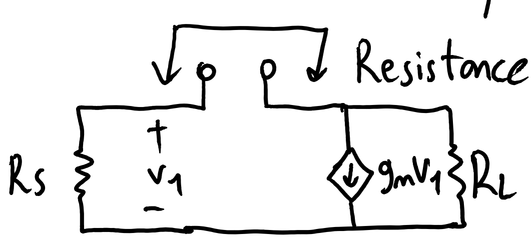
$V_L = -g_m V_1 R_L$, $V_1 = I_s R_s$ since there is no current through Rs

$V_L = -g_m R_L I_s R_s$

KVL: $V_c + I_s R_s - V_L = 0 \rightarrow V_c = -I_s R_s - g_m R_L I_s R_s$

$$\boxed{V_c(\infty) = -I_s R_s (g_m R_L + 1) = V_f}$$

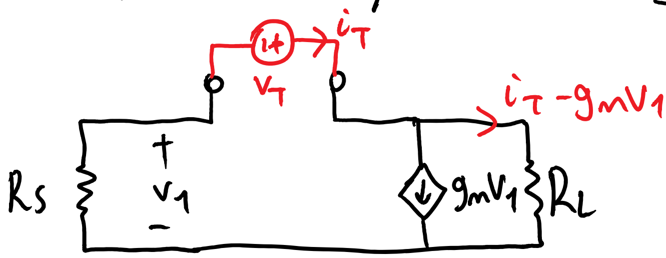
Time constant: Need to find resistance "seen" by the capacitor. Then $\tau = R_{eq} C$



Resistance "seen" by the capacitor

Remember: Kill independent sources. Have to keep dependent sources.

Method: apply a test voltage, look at how much current is drawn by the circuit. Resistance looking into that terminal is equal to $\frac{\text{test voltage}}{\text{current drawn}} = \frac{V_T}{i_T}$



$$V_1 = -i_T R_S$$

KVL: $(i_T - g_m V_1)R_L - V_1 - V_T = 0$, $i_T R_L + g_m i_T R_S R_L + i_T R_S = V_T$

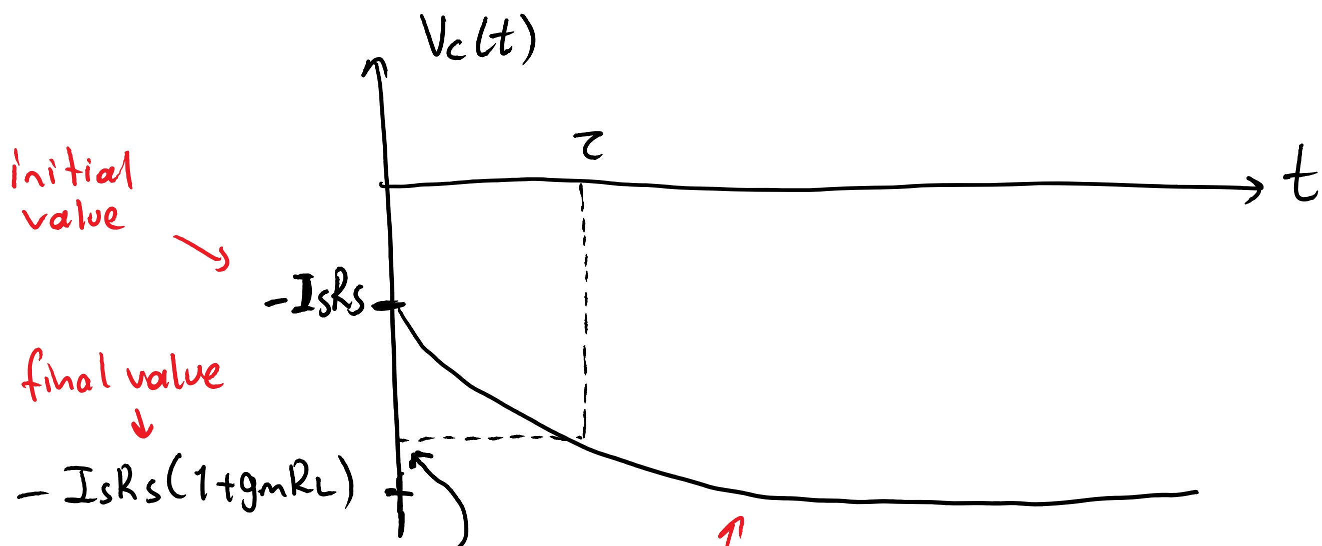
$$\rightarrow i_T(R_L + R_S + g_m R_L R_S) = V_T$$

$$\rightarrow R_{eq} = R_L + R_S + g_m R_L R_S$$

$$\boxed{\tau = (R_L + R_S + g_m R_L R_S)C}$$

Capacitor equation: $V_C(t) = V_f + (V_i - V_f)e^{-t/\tau}$

$$V_C(t) = -I_S R_S(1 + g_m R_L) + (I_S R_S g_m R_L)e^{-t/\tau}$$



exponential decay with time constant τ

$$\underbrace{I_S R_S g_m R_L\left(\frac{1}{e} - 1\right) - I_S R_S}$$

$$\cong -I_S R_S g_m R_L \times 0.632 \quad (\text{assuming } g_m R_L >> 1)$$