$V_{o_1} \equiv$ output 1 (left op amp)

P1

KCL's: $-\dfrac{V_{in}}{R_1} - \alpha V_x - \dfrac{V_{o_1}}{R_2} = 0$ ①

ideal op amps:

$V_{n_1} = V_{p_1} = 0$

$V_{o_1} = V_{p_2} = V_{n_2}$

(no current through R_5)

$\dfrac{V_{o_1} - V_{out}}{R_4} + \dfrac{V_{o_1}}{R_3} = 0$ ②

② $\rightarrow$ $V_{o_1} = V_{out} \dfrac{1}{R_4}\left(\dfrac{1}{1/R_4 + 1/R_3}\right)$

$V_x = V_{out} - V_{o_1}$

② $\rightarrow$ ①: $0 = -\dfrac{V_{in}}{R_1} - \alpha V_{out} + V_{o_1}\left(\alpha - \dfrac{1}{R_2}\right)$

$0 = -\dfrac{V_{in}}{R_1} - \alpha V_{out} + \dfrac{V_{out}}{R_4}\dfrac{1}{1/R_4 + 1/R_3}\left(\alpha - \dfrac{1}{R_2}\right)$

$\rightarrow \dfrac{V_{out}}{V_{in}} = \dfrac{1}{R_1}\left[-\alpha + \dfrac{1}{1 + R_4/R_3}\left(\alpha - \dfrac{1}{R_2}\right)\right]^{-1}$

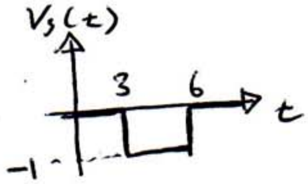
$\therefore$ $\boxed{\text{gain} = \dfrac{1}{R_1}\left[-\alpha + \dfrac{R_3}{R_3 + R_4}\left(\alpha - \dfrac{1}{R_2}\right)\right]^{-1}}$

or simplify

$\boxed{\text{gain} = -\dfrac{R_2(R_3 + R_4)}{R_1 R_3 + \alpha R_1 R_2 R_4}}$

P2

a



$-\infty < t < 3$: $V_c(t) = 0$

$3 < t < 6$: $V_c(3) = 0$ $V_c(\infty) = -1$

$$V_c(t) = e^{-(t-3)/R_1C} - 1$$

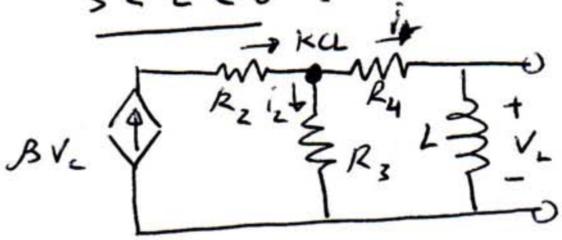
$6 < t$: $V(6) = e^{-3/R_1C} - 1$

$$V_c(t) = V_c(6)\, e^{-(t-6)/R_1C} = \left(e^{-3/R_1C} - 1\right) e^{-(t-6)/R_1C}$$

$$\therefore \boxed{V_c(t) = \begin{cases} 0 & t<3 \\ e^{-(t-3)/R_1C} - 1 & \text{for } 3<t<6 \\ \left(e^{-3/R_1C} - 1\right) e^{-(t-6)/R_1C} & 6<t \end{cases}}$$

b $-\infty < t < 3$: $V_L(t) = 0$ $V_L = L\dfrac{di_L}{dt}$

$3 < t < 6$:



$$\beta V_c = i_L + i_2 = i_L + \frac{\left(i_L R_4 + L\frac{di_L}{dt}\right)}{R_3}$$

$$= i_L\left(\frac{R_3 + R_4}{R_3}\right) + \frac{L}{R_3}\frac{di_L}{dt}$$

$$\longrightarrow i_L\left(\frac{R_3 + R_4}{L}\right) + \frac{di_L}{dt} = \frac{R_3}{L}\beta V_c$$

for $3<t<6$,

$$\therefore \underbrace{i_L\left(\frac{R_3+R_4}{L}\right)}_{k} + \frac{di_L}{dt} = \underbrace{\frac{R_3}{L}\beta}_{kA}\left[e^{\underbrace{-(t-3)/R_1C}_{m}} - 1\right]$$

$$\longrightarrow i_L = \frac{R_3\beta}{R_3 + R_4 - \frac{L}{R_1C}}\left[e^{-(t-3)/R_1C} - 1\right]$$

$$V_L = L\frac{di_L}{dt} = \frac{R_3 L \beta}{R_3 + R_4 - L/R_1 C}\left(-\frac{1}{R_1 C}\right) e^{-(t-3)/R_1 C}$$

$$V_L(t) = \frac{-R_3 L \beta}{R_1 C(R_3+R_4) - L}\, e^{-(t-3)/R_1 C}$$

$6 < t$:

$$i_L \underbrace{\left(\frac{R_3+R_4}{L}\right)}_{k} + \frac{di_L}{dt} = \underbrace{\frac{R_3}{L}\beta\left(e^{-3/R_1 C} - 1\right)}_{kA} e^{-(t-6)/\underbrace{R_1 C}_{m}}$$

$$\longrightarrow \quad i_L = \frac{R_3 \beta\left(e^{-3/R_1 C} - 1\right)}{R_3 + R_4 - L/R_1 C}\, e^{-(t-6)/R_1 C}$$

$$V_L(t) = \frac{L R_3 \beta\left(e^{-3/R_1 C} - 1\right)}{R_3 + R_4 - L/R_1 C}\left(-\frac{1}{R_1 C}\right) e^{-(t-6)/R_1 C}$$

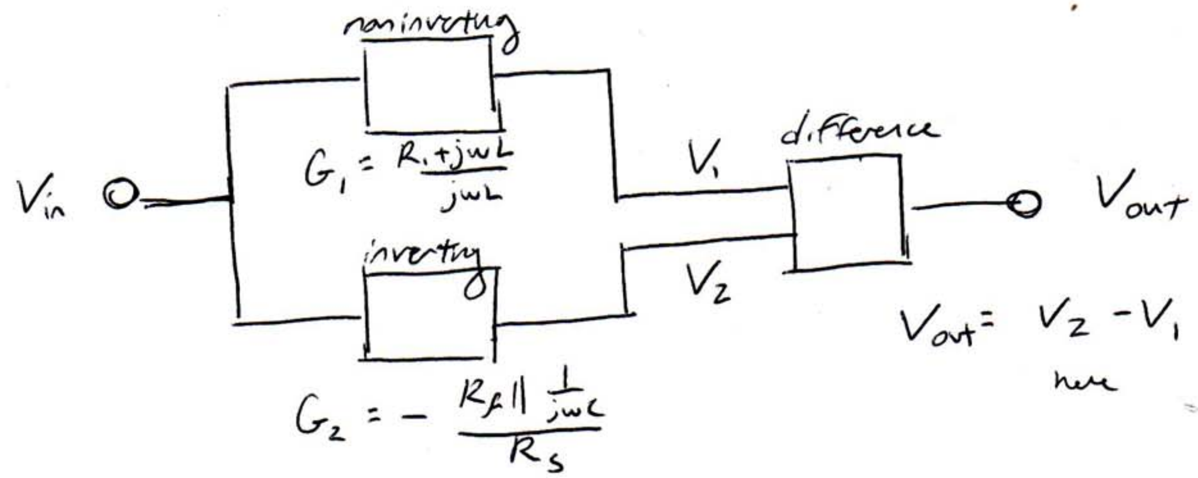
$$= -\frac{L R_3 \beta\left(e^{-3/R_1 C} - 1\right)}{R_1 C(R_3+R_4) - L}\, e^{-(t-6)/R_1 C}$$

$\therefore$

$$\boxed{V_L(t) = \begin{cases} 0 & t < 3 \\ -\dfrac{R_3 L \beta}{R_1 C(R_3+R_4) - L}\, e^{-(t-3)/R_1 C} & 3 < t < 6 \\ -\dfrac{R_3 L \beta\left(e^{-3/R_1 C} - 1\right)}{R_1 C(R_3+R_4) - L}\, e^{-(t-6)/R_1 C} & 6 < t \end{cases}}$$

P3

Interpret as a network of transfer functions:



$$\therefore \quad V_{out} = V_2 - V_1 = \left(-\frac{R_f}{R_s} \frac{1}{(1 + j\omega R_f C_f)} - \frac{R_1 + j\omega L}{j\omega L} \right) V_{in}$$

$$\boxed{H(\omega) = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_s} \frac{1}{(1 + j\omega R_f C_f)} - \frac{R_1}{j\omega L} - 1}$$

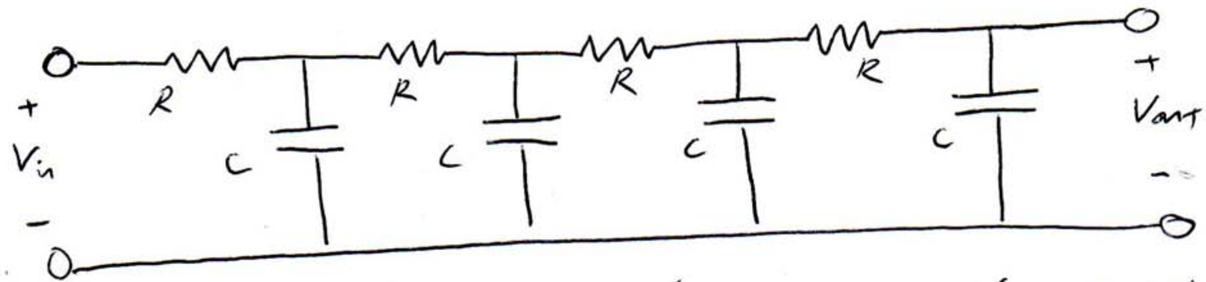
P4

(a) multiple implementations exist.... here's one:

4x RC LPF
4x RL LPF
2x LC LPF

$\omega_c = \frac{1}{RC}$ $N=4 \longrightarrow$

4 cascaded RC low-pass filters



(active works if no gain)

(b) $|H(\omega)| = \frac{1}{4} \longrightarrow \boxed{-12dB}$

(c) $\phi = 0 - \tan^{-1}(1) = -45°$

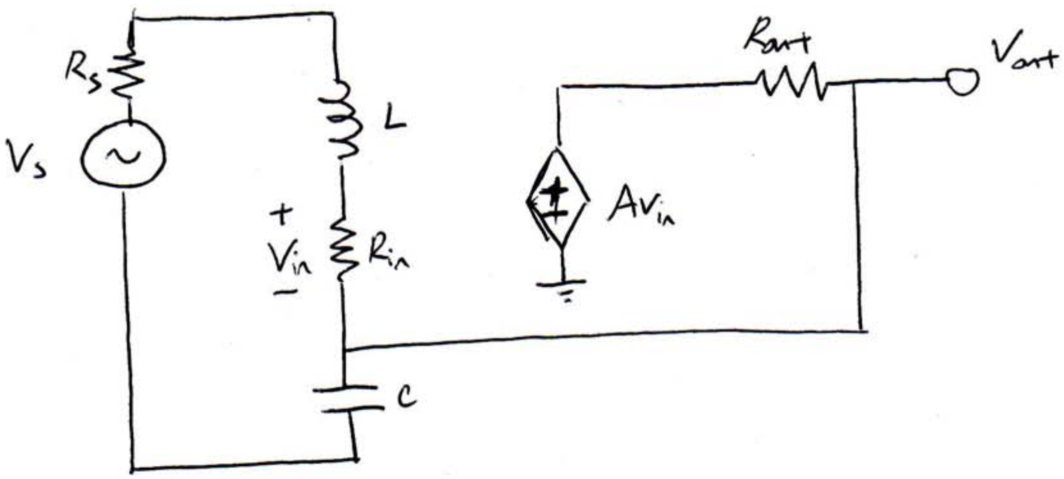
$N = 4 \longrightarrow \boxed{\phi = -180°}$

or

$\boxed{\phi = -\pi}$

P5

non-ideal



V_{out} follows phase of AV_{in} since R_{out} is purely real.

phasor domain:

$$\tilde{V}_{in} = \tilde{V}_s \frac{R_{in}}{R_s + R_{in} + j\omega L + \frac{1}{j\omega C}}$$

no phase shift b/w $\tilde{V}_s \to \tilde{V}_{in} \to \tilde{V}_{out}$ if no imaginary components.

$$\therefore \quad j\omega L + \frac{1}{j\omega C} = 0 \rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$V_s(t)$ is a sinusoidal function with this frequency.

Note: This is the resonance of an RLC circuit!