1. This is a closed book exam. However, you are allowed to bring 3 pages (8.5" x 11"), double-sided notes.

2. No electronic devices, i.e. calculators, cell phones, computers, etc.

3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.

4. **Remember to put down units. Points will be taken off for missed unit.**
Problem 1: Op Amp circuits (40 pts)

1. [20 pts] Find $V_A$ in terms of $V_1, V_2$

   Identify negative feedback since $V_A$ is fed back to negative input terminal of the 1st Opamp
   $\Rightarrow V_+ = V_-$

   Voltage at the input of 1st Opamp
   $V_+ = V_2 \times \frac{10}{10+20} = V_-$

   Current through the 10 Ohm resistor connected to $V_1$
   $I_{V1} = \frac{V_1 - V_+}{10}$

   Calculating $V_A$
   $V_A = -I_{V1} \times 10 + V_+ = \frac{2}{3} V_2 - V_1$

2. [5 pts] Find $V_B$ as a function of $V_A$

   Since no current is flowing into the input of an ideal Opamp, there is no voltage drop across the 60 Ohm resistor $\Rightarrow V_B = V_A$
3. [10 pts] Find $V_C$ as a function of $V_B$

   This is a standard non-inverting amplifier with $V_{out} = V_{in} \cdot \left(1 + \frac{R_1}{R_2}\right)$
   \[ V_C = 2 \cdot V_B \]

4. [10 pts] Find $V_O$ as a function of $V_C$

   This is a standard inverting amplifier with $V_{out} = -V_{in} \cdot \frac{R_2}{R_1}$
   \[ V_O = -2 \cdot V_C \]

5. [5 pts] Use the above to find $V_O$ in terms of $V_1$, $V_2$

   \[ V_O = -2 \cdot V_C = -4 \cdot V_B = -4 \cdot V_A = 4 \cdot V_1 - 8/3 \cdot V_2 \]
Problem 2 Diode Circuit [20 pts]
The input voltage is shown below. Assume all the diodes are ideal and with turn-on threshold voltage \( V_T = 0.6 \) V and that \( V_{in} = 5 \sin (\omega t) \)

(a) [10 pts] Draw the output voltage on the load resistor after a long time and explain how the circuit works. (The input is given as a reference.)

The shown circuit is a full wave rectifier (bridge rectifier). When the amplitude of the input voltage is high enough either \( D_1 \) and \( D_3 \) or \( D_2 \) and \( D_4 \) turn on. This means that there are always two diodes in series leading to a minimum required input voltage amplitude of \( 1.2 \) V (\( = 2 \times V_T \)) to turn them on and a voltage drop of the same amount during the on – regime. Since the current can only flow from the top to the bottom of \( R_L \) the output voltage is always positive.
(b) [10 pts] If we put a large capacitor in parallel with the load resistor at the output as shown below, what is the output voltage after a long time? Assume that, since the capacitor is large, it is not discharged significantly through $R_L$ during one period of the input signal. Draw the output you got in (a) as well for a reference and explain your answer.

The diodes allow the capacitor at the output to charge up to the maximum value of the output voltage ($V_{in} - 1.2\, V$). At the same time they prevent the capacitor to be discharged by the input source. So the only remaining discharge path would be through $R_L$. However, since $C$ is large enough that it does not get discharged significantly within one period the output voltage stays more or less constant. The minor drop in output voltage one could theoretically observe within half a period would be compensated for by the second diode pair to turn on and charge up the capacitor to the maximum voltage level again.
Problem 3. [30 pts] Semiconductor Device Physics
Consider a p-i-n diode with a charge density distribution given by the figure below. The permittivity of the material is \( \varepsilon \). Further \( N_1 x_1 = N_2 (x_3 - x_2) \)

![Charge density diagram](image)

a) [4 pts] Which side is the p side? Which side is the n side? Motivate your answer.

The right side (from \( x_2 \) to \( x_3 \)) is the n side. The left side (-\( x_1 \) to 0) is the p side. Positive charge density implies donor atom doping since donors become positively charged ions (similarly, a negative charge density on the left side implies acceptor atom doping).

b) [2 pts] What is the direction of the built-in electric field?

Integral of charge density is negative from \(-x_1\) to \(x_3\). This implies the electric field goes from right to left.

c) [4 pts] What is the value for the electric field \( E(x) \) for \( x \leq -x_1 \)? What about \( x \geq x_3 \)?

\[ E(x) = 0 \text{ for } x \leq -x_1 \text{ and also for } x \geq x_3 \]

d) [10 pts] What is \( E(x) \) for \(-x_1 \leq x \leq x_3\)?

Give your answer in functions of \( x \) for the different regions of \( x \).

\[
\text{Gauss' Law: } E(x) = \frac{1}{\varepsilon} \int \rho(x) \, dx \\
E(x) = -\frac{q}{\varepsilon} N_1 (x + x_1) \text{ for } -x_1 \leq x \leq 0 \\
E(x) = -\frac{q}{\varepsilon} N_1 x_1 \text{ for } 0 \leq x \leq x_2 \\
E(x) = \left( -\frac{q}{\varepsilon} N_1 x_1 \right) + \left( \frac{q}{\varepsilon} N_2 (x - x_2) \right) \text{ for } x_2 \leq x \leq x_3
\]
e) [4 pts] What is the range of $x$ for which the magnitude of the E-field is maximum?

The magnitude of the E-field is maximum (most negative) for $0 \leq x \leq x_2$.

f) [6 pts] Sketch $E(x)$ vs. $x$. Label your plot.
Problem 4. [10 pts]. Load-line Analysis

Consider the following circuit. The I-V characteristics of the boxed element is given in the graph next to the circuit.
What is (a) $V_D$ and (b) $I_D$?

$V_D = 1\text{V}, I_D = 1\text{mA}$