1. This is a closed book exam. However, you are allowed to bring 4 pages (8.5” x 11”), double-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. Remember to put down units. Points will be taken off for missed unit.
Problem 1: Transistor Amplifier (116 pts)

Note: The sub-parts are somewhat independent. Read through the entire problem even if you are not able to solve some parts. Good luck!

In this problem you are about to analyze the amplifier circuit shown in the figure below. The threshold voltage $V_{t0} = 1$ V and the capacitor impedance at signal frequency is negligible.

The diodes D in figure 1 have the following I-V characteristic:

![Figure 1: Amplifier circuit for Problem 1](image1)

![Figure 2: I-V characteristics of diodes in circuit shown in figure 1.](image2)
a) Derive an expression for $V_{\text{diode}}$ [V] as a function of $I_{\text{diode}}$ [A] for the “on-region” of the diode and give an equivalent circuit that has the same I-V characteristic in that region (10 pts)

$$V_{\text{diode}} = 0.7 + 300 \times I_{\text{diode}}$$

Equivalent circuit: A 0.7 V voltage source in series with a 300 Ohm resistor

b) Calculate $V_G$, the DC voltage at the gate of the MOS transistor (15 pts).

We basically have a resistive voltage divider between 5 V and 3 * 0.7 V, with resistors of 2000 Ohm and 3 x 300 Ohm

$$V_G = 2.1 + (5-2.1)*900/(2000 + 900) = 2.1 + 2.9/2900 * 900 = 3 \text{ V}$$

Alternatively once can solve the following equation for the current through the 3 diodes in series

$$I_{\text{diodes}} = (5 – 3 * V_{\text{diode}})/2000 = (5 – 3*(0.7 + 300 I_{\text{diodes}}))/2000$$

$$V_G = 5 – 2000*0.001 = 3 \text{ V}$$

c) Express $V_{GS}$, the DC voltage between the gate and the source of the transistor, as a function of $I_{DS}$. (4 pts)

$$V_{GS} = V_G - V_{\text{diode}} = 3 – 0.7 – 300 \times I_{DS} = 2.3 – 300 \times I_{DS}$$
d) Assume that for a similar circuit you found out that

\[ V_{GS} = 2.4 - I_{DS} \times 200. \]

Using the I-V characteristic for the transistor in saturation (given below) and Load-Line analysis find \( V_{GSQ} \) and \( I_{DSQ} \) (12 pts)

(Note that the given \( V_{GS} \) vs. \( I_{DS} \) relation is not necessarily equal to what you found in part c). This is a new starting point for the problem in case you encountered some problems in parts a) – c))

![VGS vs. IDS for MOS Transistor](image)

Figure 3: \( I_{DS} \) vs. \( V_{GS} \) for the transistor used in circuit in Figure 1 (in saturation)

From the load-line analysis above we found \( I_{DS} = 2 \text{mA} \) and \( V_{GS} = 2 \text{V} \)
e) For the values found in d) and given in the problem, in Figure 1 and the I-V characteristic of the diode: Is the transistor in saturation? Why or why not? (12 pts)

If $I_{DS}$ is 2mA the voltage drop across 1kOhm resistor is 2 V. From the diode characteristics the voltage drop across the diode is 1.3 V. That leaves 5 – 3.3 V for $V_{DS}$ which is 1.7 V. We have to check whether $V_{DS} > V_{GS} - V_T$ and since 1.7 > 2 – 1 V the transistor is in saturation.

If one argues that $V_G$ is 3 V and since $V_{GS}$ is equal to 2 V the voltage drop across the diode has to be 1 V it is ok as well. In that case $V_{DS}$ would be 2V and the transistor still in saturation.

f) Based on the values you got in part d), calculate $g_m$ (K of transistor is 2mA/V$^2$) (assume the transistor is in saturation regardless of your answer in part e) ) (4 pts)

$$g_m = 2 \sqrt{K \cdot I_{DS}} = 4 \text{ mS}$$
g) Draw the small signal equivalent circuit for the circuit given in Figure 1. You can neglect \( r_d \) of the transistor. If you don’t know how to deal with the diodes simply replace them with 100 Ω resistors in the small signal circuit (for partial credit). If you include the full diode model, explain how you do it. (27 pts if model includes diodes, 24 pts if you choose to replace diodes with 100 Ω resistors)

Diodes can be simply replaced by their equivalent resistance in the on region. The DC voltage sources of the diode model are replaced by a short. Values in parenthesis are values if 100 Ohm resistor used in stead of diodes.
h) For this part of the problem assume a $g_m$ of 10 mS (again this might be different from the $g_m$ you found in part f))

Using small signal analysis and the circuit you drew in g), find the small signal voltage gain $A_v = \frac{V_{out}}{V_{in}}$ and the input impedance of the circuit without the AC source and the 150 Ω resistor connected to $V_{in}$ (you can leave the numerical results in fractions).

(24 pts)

\[
V_{out} = -g_m \cdot 1 \text{ kOhm} \\
V_{GS} = V_{in} - g_m \cdot V_{GS} \cdot 300 \Rightarrow V_{gs} = V_{in} / (1 + 300 \cdot g_m) \\
\Rightarrow V_{out} = -g_m \cdot 1 \text{ kOhm} \cdot V_{in} / (1+300 \cdot g_m) \\
\Rightarrow A_v = 10 / (1+3) = -2.5 (-5 if 100 Ohm resistors are used instead of diodes)
\]

$R_{in} = \frac{V_{in}}{I_{in}} = 2k\Omega \text{//}900k\Omega$ is approx. 600 Ohm ($=\frac{1800000}{2900}$)
(6000000/2300 or approx 300 Ohm if 100 Ohm resistor used instead of diodes)

i) Now connect the AC source and the 150 Ω resistance to the circuit again (as shown in figure 1) (8 pts)

Calculate the AC output voltage $v_{out}$ for a source voltage amplitude $v_s$ of 0.1 V.

\[
V_{out} = v_s \cdot R_{in} / (150 + R_{in}) \cdot A_v = 0.1 \cdot 4/5 \cdot 2.5 = 0.2 \text{ V} \ (0.33 \text{ V if 100 Ohm resistor is used})
\]
Problem 2: Logic Circuits (36 pts)

a) Complete the truth tables for the following circuits. Assume $V_{DD} = 5$ V, diodes are ideal ($V_{on} = 0$ V), the transistors (all NMOS) have a $V_{to}$ of 1 V, a logic 0 at the input is 0 V, a logic 1 at the input is $V_{DD}$ and any voltage above (below) $V_{DD} / 2$ at the output is considered a logic 1 (logic 0). (24 pts)
b) For this part assume any diode you use has an on-voltage of 0.6 V, but is otherwise ideal. Further assume that any transistor you use has a non-zero on-resistance but an infinite off-resistance. $V_{t0}$ of the transistors are 1 V and $V_{DD}$ is 5 V

Design a 2-input OR gate with an output high level equal to $V_{DD}$ and an output low level being equal to GND using any device (out of R, L, C, Diodes, NMOS, PMOS Transistors) and as many devices you want. Explain why your circuit fulfills the requirements. (12 pts)

There are a couple of possible solutions:

- e.g. implementing a NOR (CMOS or transistor-resistor logic) gate and then connecting a CMOS inverter to its output.
- Or an OR gate in diode logic and then connecting 2 inverters in series to its output to make sure the output levels are 0 and VDD for low and high respectively
Problem 3: Diode Circuits (60 pts)
In this problem you are to analyze the circuit below. Assume a forward bias turn-on threshold voltage for each diode (Zener as well as regular diode) of 0.6 V. Further assume that the resistors R are big enough such that the capacitor C does not charge nor discharge noticeably through them within a period of $V_{in}$.

If you don’t know how to solve the problem with both Zener diodes, you can replace the 100 V Zener diode with a regular diode (for partial credit). Clearly state that you do so though!

Using the 100 V Zener diode or a regular diode does not make a difference here since we never reach 100 V here

a) What is $V_x$ and $V_{out}$ if $V_{in} = 0$ V? (6 pts; (4 pts if regular instead of 100 V Zener diode is used)

$V_x = 4.4 + 0.6 = 5$ V (4.4 V Zener diode in reverse breakdown, 100 V Zener diode forward biased)

$V_{out} = V_x - 0.6 = 4.4$ V
b) Now assume that $V_{in}$ has been 0 for a while before it gets turned on at $t=0$ and has an amplitude of 2 V. $V_{in}$ is given in the graph below. Complete the graph with $V_C$, $V_x$, and $V_{out}$. Make sure you label your graph! (Derive $V_C$ at $t=0$ from part a)) (24 pts)

$V_C = -5$ V at the beginning but charges to -3 V within the first quarter of T (since $V_x$ is clamped to 5 V and $V_{in}$ raises to +2 V $\rightarrow$ Capacitor charges). From there on, since $V_x$ is never larger than 5 V again, the zener diode branch is basically always off and the voltage across the capacitor cannot change anymore (no charge/discharge path) and $V_x = V_{in} - V_C$ for $t > T/4$. $V_{out}$ is always $V_x - 0.6$ V due to the voltage drop across the diode.
c) Repeat part b), but now with an $V_{in} = 3\ V$ (28 pts)

$V_C = -5\ V$ at the beginning but charges to $-2\ V$ within the first quarter of $T$ (since $V_x$ is clamped to $5\ V$ and $V_{in}$ raises to $+3\ V \rightarrow$ Capacitor charges). From there on, since $V_x$ is never larger than $5\ V$ again, the zener diode branch is basically always off and the voltage across the capacitor cannot change anymore (no charge/discharge path) and $V_x = V_{in} - V_C$ for $t > T/4$. $V_{out}$ is $V_x - 0.6\ V$ due to the voltage drop across the diode if $V_x > 0.6\ V$ and zero otherwise (after all it is a simple half-wave rectifier)
Problem 4: Transient Analysis (90 pts)

The following circuit is given

The switch has been closed (connected to the left) for a long time but at T=0, the switch is flipped

a. What are $V_A$, $V_B$, $I_A$, $I_B$, and $I_L$ at time T=0? (10 pts)

$V_A = 1 \text{ V (capacitor is an open circuit at DC)}$

$V_B = 5 \times \frac{1}{4} = 1.25 \text{ V (voltage divider between 1 & 3 } \Omega \text{ resistor, inductor is a short at DC)}$

$I_A = 0 \text{ A (capacitor is an open circuit at DC)}$

$I_B = 1.25 / 1 = 1.25 \text{ A (Ohm’s Law)}$

$I_L = I_B = 1.25 \text{ A (KCL)}$
b. Immediately after the switch has been flipped:
What are $V_A$, $V_B$, $I_A$, $I_B$ and $I_L$ the current through the inductor at time $T=0^+$ (after the switch has been flipped) (10 pts)

$V_A = 1 \text{ V} \text{ (voltage at capacitor cannot change instantaneously)}$
$V_B = 1 \text{ V} \text{ (VA and VB are the same now)}$
$I_A = I_L - I_B = 0.25 \text{ A} \text{ (current through inductor cannot change instantaneously)}$
$I_B = 1 / 1 = 1 \text{ A} \text{ (Ohm’s Law)}$
$I_L = 1.25 \text{ A} \text{ (current through inductor cannot change instantaneously)}$

c. What are $V_A$, $V_B$, $I_A$, $I_B$ after a long time ($T = \infty$) (8 pts)

$V_A = 1.25 \text{ V} \text{ (capacitor is an open circuit at DC)}$
$V_B = 5 * ¼ = 1.25 \text{ V} \text{ (voltage divider between 1 & 3 \Omega resistor, inductor is a short at DC, capacitor an open circuit at DC)}$
$I_A = 0 \text{ A} \text{ (capacitor is an open circuit at DC)}$
$I_B = 1.25 / 1 = 1.25 \text{ A} \text{ (Ohm’s Law)}$
d. Write the differential equation to find $V_A$ (30 pts)

\[ V_2 = l_{\text{total}} + 3 + V_L + V_A = 3 \cdot l_{\text{total}} + L \frac{dl_{\text{total}}}{dt} + V_A = 5 = 3 \cdot l_{\text{total}} + 0.5 \frac{dl_{\text{total}}}{dt} + V_A \]

\[ l_{\text{total}} = I_A + I_B = C \frac{dV_A}{dt} + V_A / 1 = 0.5 \frac{dV_A}{dt} + V_A \]

Plugging in $l_{\text{total}}$ in first equation

\[ 5 = 3 \cdot V_A + 1.5 \frac{dV_A}{dt} + 0.25 \frac{d^2V_A}{dt^2} + 0.5 \frac{dV_A}{dt} + V_A \]

\[ 20 = \frac{d^2V_A}{dt^2} + 8 \frac{dV_A}{dt} + 16 V_A \]
e. Solving for $V_A$

i. Give the forced response for $V_A$ (4 pts)

$$V_A = 1.25 \text{ V (DC solution)}$$

ii. Find the natural response for $V_A$ (9 pts)

Damping ratio can be found from differential equation found in part f

$\zeta = 1$

$\rightarrow$ Critical damping

$\rightarrow$ Natural response: $V_A (t) = K_1 e^{-4t} + K_2 \cdot t \cdot e^{-4t}$

iii. Using the above and the initial conditions given, find the complete solution for $V_A$ (9 pts)

$$V_A (t) = K_1 e^{-4t} + K_2 \cdot t \cdot e^{-4t} + 1.25$$

$V_A(t=0) = 1 \text{ V } K_1 + 1.25 \rightarrow K_1 = -0.25 \text{ V}$

$I_A(t=0) = 0.25 = C \cdot \frac{dV_A}{dt} = 0.5 \cdot (-4K_1 + K_2) = 0.5 + 0.5K_2 \rightarrow K_2 = -0.5 \text{ V}$

$$V_A (t) = -0.25 e^{-4t} + -0.5 \text{ V } t \cdot e^{-4t} + 1.25$$
f. After a long time we flip the switch back

   i. What happens to the capacitor and the voltage source $V_1$? (5 pts)

   Right after the switch gets flipped back $V_C$ stays at 1.25 V and the capacitor starts getting discharged through the 10 kOhm resistor to 1 V. During that time current flows from the capacitor to the voltage source. It basically gets “charged”

   ii. What happens to the inductor and the voltage source $V_2$? (5 pts)

   Nothing! Removing the charged capacitor does not change anything to the circuit on the right, since the capacitor was not drawing any current anymore anyway