Problem 1: Circuit Analysis [15 points in total]

a) In the circuit below, the independent source values and resistances are known. Use the nodal analysis method to write 2 equations sufficient to solve for $I_a$ and $I_b$. To receive credit, you must write your answer in the box below. [5 pts]

DO NOT SOLVE THE EQUATIONS!

KCL:
The algebraic sum of all currents flowing into a node is zero.

Applying KCL to node a:

\[
\frac{V_{AA} - V_a}{R_1} + I_{AA} - I_{bb} + \frac{V_b - V_a}{R_3} = 0
\]

Applying KCL to node b:

\[
\frac{V_a - V_b}{R_3} + I_{bb} + \frac{-V_{bb} - V_b}{R_4} = 0
\]

Write your equations here:

\[
\frac{V_{AA} - V_a}{R_1} + I_{AA} - I_{bb} + \frac{V_b - V_a}{R_3} = 0
\]

\[
\frac{V_a - V_b}{R_3} + I_{bb} - \frac{V_{bb} + V_b}{R_4} = 0
\]
b) In the circuit below, the independent source values and resistances are known. Use the node-voltage method to write 2 equations sufficient to solve for $V_a$ and $V_b$. To receive credit, you must write your answer in the box below. [5 pts]

DO NOT SOLVE THE EQUATIONS!

\[
V_{AA} - V_a + \frac{0-V_a}{R_1} + \frac{0-V_b}{R_2} - I_{BB} = 0
\]

Constraint imposed by floating voltage source:

\[
V_b - V_A = V_{BB}
\]

Write your equations here

\[
\begin{align*}
\frac{V_{AA} - V_a}{R_1} - \frac{V_a}{R_2} - \frac{V_b}{R_3} - I_{BB} &= 0 \\
V_b - V_A &= V_{BB}
\end{align*}
\]

e) Find $V_v$ [5 pts]

\[
\begin{align*}
\frac{20 - V_v}{5} &= i_x \\
i_x &= \frac{20 - V_v}{5}
\end{align*}
\]

Applying KCL to node $x$:

\[
\begin{align*}
i_x - 0.5i_x &= \frac{V_v - 0}{10} = 0 \\
0.5i_x &= \frac{V_v}{10} = 0 \\
i_x &= \frac{V_v}{5} = 0 \\
20 - V_v &= \frac{V_v}{5} = 0 \\
20 - V_v &= V_v = 0 \\
20 &= 2V_v \\
10 &= V_v
\end{align*}
\]

\[V_v = 10 \text{ Volts}\]
Problem 2: Equivalent Circuits [21 points in total]

ii) What is the maximum resistance which can be achieved by connecting these five resistors? Show how they should be connected in this case. [13 pts]

Connect the resistors in series, in order to achieve the largest equivalent resistance. 

\[ R_{eq} = \sum R_i = 50 \text{ k}\Omega \]

Circuit diagram of resistors connected to give a maximum resistance value of \(50 \text{ k}\Omega\):

\[ \begin{array}{c}
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega
\end{array} \]

ii) What is the minimum resistance which can be achieved by connecting these five resistors? Show how they should be connected in this case. [13 pts]

\[ \frac{1}{R_{eq}} = \frac{1}{\sum \frac{1}{R_i}} = \frac{1}{5(\frac{1}{10k\Omega})} = \frac{1}{2k\Omega} \]

Circuit diagram of resistors connected to give a minimum resistance value of \(2 \text{ k}\Omega\):

\[ \begin{array}{c}
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega \\
\hline
10k\Omega
\end{array} \]
ii) The I-V characteristic of a linear circuit is given below. Find the Thévenin equivalent of this circuit. [3 pts]

When \( i = 0 \), \( V = V_{Th} \): From the I-V plot, \( V = -2V \) when \( i = 0 \).
Therefore, \( V_{Th} = -2V \)

When \( V = 0 \), \( i = -V_{Th} / R_{Th} \):

\[
V_{Th} = -2V
\]

\[
i = 1mA \text{ when } V = 0.
\]

\[
i_{th} = 1mA \text{ at } V = 0.
\]

From the I-V plot:

\[
V_{Th} = -2V
\]

\[
r_{th} = 2k\Omega
\]

From the maximum power transfer theorem, \( R_L = R_{Th} \), for maximum power to be absorbed by \( R_L \).

\[
r_{th} = 0.5mW
\]

Power delivered to load resistor:

\[
I^2R_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L = \frac{(-2)^2}{4(2k\Omega)} = \frac{1}{2} mW
\]

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Problem 3: Op Amp Circuit [14 points in total]
Consider the op amp circuit below:

\[
\text{ideal op amp:}
\]

\[
V_{in} = V_p = 0V
\]

\[
i_{in} = 0A
\]

\[
i_p = 0A
\]

---

a) Assuming the op amp is operating in its linear region, find an expression for \( v_{out} \) as a function of \( v_{in} \). [10 pts]

Applying KCL to inverting input node:

\[
\frac{V_{in} - 0}{1k\Omega} + \frac{V_{x} - 0}{2k\Omega} = 0
\]

\[
\Rightarrow V_{x} = -2V_{in}
\]

Applying KCL to node X:

\[
\frac{0 - V_x}{2k\Omega} + \frac{0 - V_x}{2k\Omega} + \frac{V_{out} - V_x}{2k\Omega} = 0
\]

\[
\Rightarrow V_{out} = 2V_x = 2(-2V_{in})
\]

\[
v_{out} = -6V_{in}
\]

b) Sketch the voltage transfer characteristic for the op amp circuit, for \( v_{in} \) ranging from 0 Volts to -5 Volts. Indicate the minimum and maximum values of \( v_{out} \) [4 pts]

Due to the positive and negative power supply voltages, 

\[-12V \leq v_{out} \leq 12V\]