EECS 20. Solution to Midterm 1. 5 March 1999

1. **24 points** Fill in the blanks:

   (a) If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, then $A \cap B = \{3\}$ and $A \cup B = \{1, 2, 3, 4, 5\}$

   (b) If the predicates P, Q, R all evaluate to true then the predicate $[\neg P \wedge \neg Q] \vee (\neg R)$ evaluates to *false*.

   (c) If $f : X \to Y$ and $g : Y \to X$, then $f \circ g : Y \to Y$ and $g \circ f : X \to X$

2. **24 points**
   Fill in the blanks:

   (a) Euler's formula is $\exp i\theta = \cos(\theta) + i \sin(\theta)$.

   (b) The four distinct roots of the equation $z^4 = 1$ are $1, -1, i, -i$.

   (c) If $\forall t \in Reals \quad \cos(t + \pi/2) = Re[A \exp(jt)]$ then $A = j$.

3. **20 points** Find the period and the fundamental frequency of the following functions and then give their Fourier series representation as a sum of complex exponentials. All functions are denoted $x : Time \to Comps$, and time is in seconds.

   (a) $\forall t \in Time \quad x(t) = 10 \sin(2\pi t) + 10 \cos(2\pi t)$
   $= (5 - 5j)e^{j2\pi t} + (5 + 5j)e^{-j2\pi t}$.
   The fundamental frequency is 1 Hz and the fundamental period is 1 sec.

   (b) $\forall t \in Time \quad x(t) = (1 + i) \sin(2\pi t) + \sin(\pi t)$
   $= -i/2 \times e^{i\pi t} + i/2 \times e^{-i\pi t} + (1 - i)/2 \times e^{i2\pi t} - (1 - i)/2 \times e^{-i2\pi t}$.
   The fundemental frequency is 0.5 Hz and the fundamental period is 2 sec.

4. **20 points** Fill in the blanks:

   (a) Let D_τ be the system that produces a delay of τ. Let $x : Reals \to Comps$. Then

   $$\forall t \quad [D_\tau(x)](t) = x(t - \tau)$$

   (b) Let H be the frequency response of the system $L = D_1 + D_2$. Then

   $$\forall \omega \in Reals \quad H(\omega) = e^{-j\omega} + e^{-2j\omega}.$$

   To obtain this result, consider an exponential input x to L,

   $$\forall t, \quad x(t) = \exp(i\omega t).$$

   Let the response be $y = L(x) = D_1(x) + D_2(x)$. Then

   $$\forall t, \quad y(t) = [e^{-i\omega} + e^{-2i\omega}] \exp(i\omega t)$$

   and so $H(\omega) = e^{-i\omega} + e^{-2i\omega}$.

5. **20 points** Suppose $L : Inputs \to Outputs$ is an LTI system, where $Inputs = Outputs = [Ints \to Reals]$. Let u denote the unit step function:

$$\forall t \in Ints \quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t = 0, 1, 2, \cdots \end{cases}$$

Suppose $y = L(u)$ is given by:

$$\forall t \in Ints \quad y(t) = \begin{cases} 0 & t < 0 \\ t & t = 0, 1, 2, \cdots \end{cases}$$

Consider the input signal z:

$$\forall t \in Ints \quad z(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Let $w = L(z)$.

(a) Plot the signals u, y, z.

(b) Determine w and plot w.

Carefully mark the appropriate time values and magnitudes in your plots.

Figure 1 gives the plots of the four signals. The plots of u, y, z are obtained directly. The plot of w is obtained as follows. We observe that $z = u - D_1(u)$ so

$$\begin{aligned} w = L(z) = L(u - D_1(u)) &= L(u) - D_1(L(u)) \quad \text{by LTI} \\ &= y - D_1(y) \end{aligned}$$

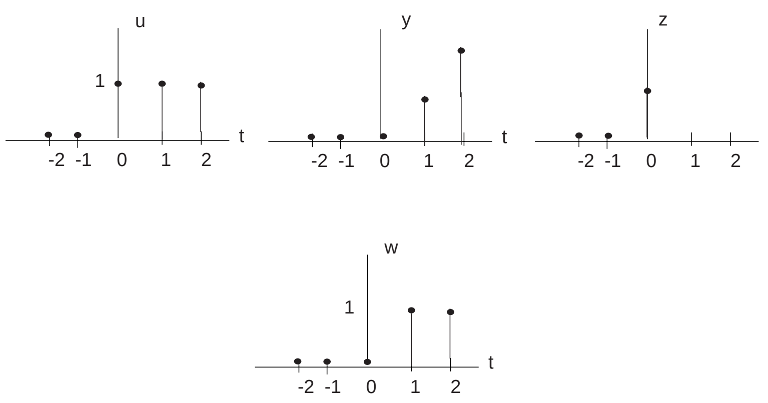

Figure 1: Figure for Problem 5