1. **24 points** Fill in the blanks:

(a) If \( A = \{1, 2, 3\} \), \( B = \{3, 4, 5\} \), then \( A \cap B = \{3\} \) and \( A \cup B = \{1, 2, 3, 4, 5\} \).

(b) If the predicates \( P, Q, R \) all evaluate to true then the predicate \( \neg P \land \neg Q \lor (\neg R) \) evaluates to false.

(c) If \( f : X \to Y \) and \( g : Y \to X \), then \( f \circ g : Y \to Y \) and \( g \circ f : X \to X \).

2. **24 points**

Fill in the blanks:

(a) Euler’s formula is \( \exp i\theta = \cos(\theta) + i\sin(\theta) \).

(b) The four distinct roots of the equation \( z^4 = 1 \) are \( 1, -1, i, -i \).

(c) If \( \forall t \in \text{Reals} \quad \cos(t + \pi/2) = \text{Re}[A \exp(jt)] \) then \( A = j \).

3. **20 points** Find the period and the fundamental frequency of the following functions and then give their Fourier series representation as a sum of complex exponentials. All functions are denoted \( x : \text{Time} \to \text{Comps} \), and time is in seconds.

(a) \( \forall t \in \text{Time} \quad x(t) = 10 \sin(2\pi t) + 10 \cos(2\pi t) = (5 - 5j)e^{2\pi t} + (5 + 5j)e^{-2\pi t} \).

The fundamental frequency is 1 Hz and the fundamental period is 1 sec.

(b) \( \forall t \in \text{Time} \quad x(t) = (1 + i) \sin(2\pi t) + \sin(\pi t) = -i/2 \times e^{i\pi t} + i/2 \times e^{-i\pi t} + (1 - i)/2 \times e^{i2\pi t} - (1 - i)/2 \times e^{-i2\pi t} \).

The fundamental frequency is 0.5 Hz and the fundamental period is 2 sec.

4. **20 points** Fill in the blanks:

(a) Let \( D_\tau \) be the system that produces a delay of \( \tau \). Let \( x : \text{Reals} \to \text{Comps} \). Then \( \forall t \quad [D_\tau(x)](t) = x(t - \tau) \).

(b) Let \( H \) be the frequency response of the system \( L = D_1 + D_2 \). Then \( \forall \omega \in \text{Reals} \quad H(\omega) = e^{-j\omega} + e^{-2j\omega} \).

To obtain this result, consider an exponential input \( x \) to \( L \),

\( \forall t, \quad x(t) = \exp(i\omega t) \).

Let the response be \( y = L(x) = D_1(x) + D_2(x) \). Then \( \forall t, \quad y(t) = [e^{-i\omega} + e^{-2i\omega}] \exp(i\omega t) \)

and so \( H(\omega) = e^{-i\omega} + e^{-2i\omega} \).
5. **20 points** Suppose $L : Inputs \rightarrow Outputs$ is an LTI system, where $Inputs = Outputs = [\text{Ints} \rightarrow \text{Reals}]$. Let $u$ denote the unit step function:

$$\forall t \in \text{Ints} \quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t = 0, 1, 2, \ldots \end{cases}$$

Suppose $y = L(u)$ is given by:

$$\forall t \in \text{Ints} \quad y(t) = \begin{cases} 0 & t < 0 \\ t & t = 0, 1, 2, \ldots \end{cases}$$

Consider the input signal $z$:

$$\forall t \in \text{Ints} \quad z(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Let $w = L(z)$.

(a) Plot the signals $u, y, z$.
(b) Determine $w$ and plot $w$.

Carefully mark the appropriate time values and magnitudes in your plots.

Figure 1 gives the plots of the four signals. The plots of $u, y, z$ are obtained directly. The plot of $w$ is obtained as follows. We observe that $z = u - D_1(u)$ so

$$w = L(z) = L(u - D_1(u)) = L(u) - D_1(L(u)) \quad \text{by LTI}$$
$$= y - D_1(y)$$

![Figure 1: Figure for Problem 5](image-url)