EECS 20n: Structure and Interpretation of Signals and Systems — Final Exam
Department of Electrical Engineering and Computer Sciences — 14 May 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________ FIRST Name ______________ Lab Time ______________

- **Please write you name and Lab Time in the spaces provided above.**
- This exam consists of a total of 20 pages, including a double-sided appendix sheet containing transform properties. When you are permitted to begin work, verify that your copy contains all the pages and that there are no printing anomalies. If you detect a problem, please notify the staff immediately.
- The exam should take you up to 3 hours to complete. We recommend that you budget your time according to the point allocation for each problem and/or part thereof.
- Please limit your work to the space provided for each problem. *No other work will be considered in grading your exam.*
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- Full credit will be given *only if* your work is clearly explained.

Problem	Points	Your Score
1	5	
2	15	
3	10	
4	15	
5	25	
6	15	
7	20	
8	10	
9	15	
Total	**130**	

- Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \ ,$$

  where $p = \frac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable contiguous discrete interval of length p (for example, $\sum_{k=\langle p \rangle}$ can denote $\sum_{k=0}^{p-1}$).

- Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period p:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ik\omega_0 t} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt \ ,$$

  where $p = \frac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable continuous interval of length p (for example, $\int_{\langle p \rangle}$ can denote $\int_0^p$).

- Discrete-time Fourier transform (DTFT) synthesis and analysis equations for a discrete-time signal:

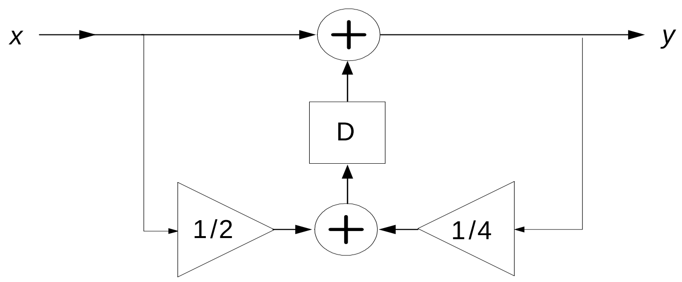
$$x(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\omega) e^{i\omega n} d\omega \qquad \longleftrightarrow \qquad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n} \ ,$$

  where $\langle 2\pi \rangle$ denotes a suitable continuous interval of length 2π (for example, $\int_{\langle 2\pi \rangle}$ can denote $\int_0^{2\pi}$ or $\int_{-\pi}^{\pi}$).

- Continuous-time Fourier transform (CTFT) synthesis and analysis equations for a continuous-time signal:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \qquad \longleftrightarrow \qquad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \ .$$

Problem 1 (5 Points Total) A causal system is initially at rest (i.e., all initial conditions are zero) and is described by the following delay-adder-gain block diagram. The block D denotes a delay by one sample; that is, if the input to the delay block D is a signal p, the output of the delay block is q, where $q(n) = p(n-1)$ for all n.



Is the system with the input x and the output y LTI? Explain your reasoning.

Problem 2 (15 Points Total) For each signal in parts (a) and (b) below, state whether the signal has a complex exponential Fourier series representation and explain your reasoning. If the signal does have a Fourier series representation, determine all the corresponding Fourier series coefficients.

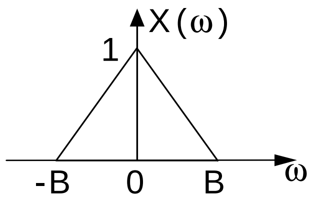
(a) (10 Points) The discrete-time signal $q(n) = \cos\left(\frac{2\pi n}{3}\right) + \cos(\pi n)$.

(b) (5 Points) The continuous-time signal $v(t) = \cos(\pi t) + \cos(t)$.

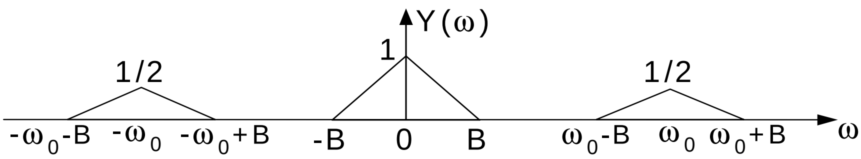
Hint: What is the period p of each signal?

Problem 3 (10 Points Total) Consider the continuous-time system F shown below.

The input signal x, has the following continuous-time Fourier transform:



The output signal y corresponding to the above input has the following continuous-time Fourier transform:

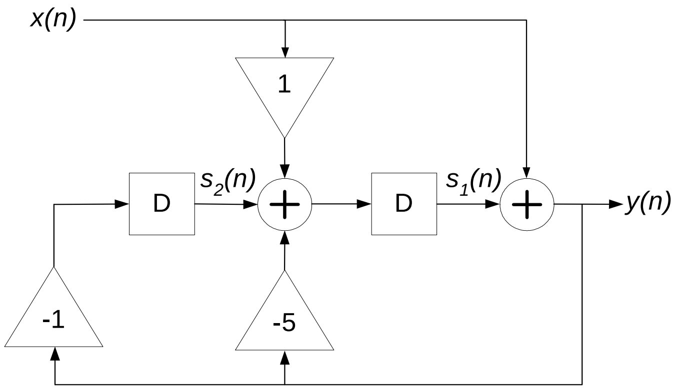


You should assume that $X(\omega)$ and $Y(\omega)$ are zero outside the ranges indicated.

(a) (5 Points) Can the system F be linear? If so, give an example of such an F. If not, explain why.

(b) (5 Points) Can the system F be LTI? If so, give an example of such an F. If not, explain why.

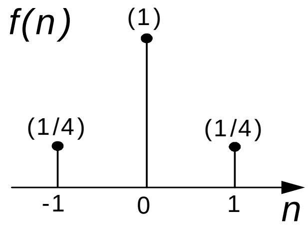
Problem 4 (15 Points Total) A real, causal, discrete-time LTI system is characterized by the following delay-adder-gain block diagram. The input and output at time n are denoted by $x(n)$ and $y(n)$, respectively. Each block D corresponds to a delay by one sample; that is, if the input to the delay block D is a signal p, the output of the delay block is q, where $q(n) = p(n-1)$ for all n.



(a) (7 Points) Determine the linear, constant-coefficient difference equation that describes the relationship between the input x and the output y.

(b) (8 Points) The outputs of the delay blocks are selected as the state variables $s_1(n)$ and $s_2(n)$ (see the figure above). For this selection of state variables, determine the corresponding state-space $[A, b, c, d]$ representation of the LTI system.

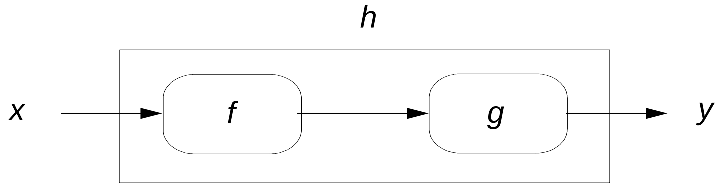
Problem 5 (25 Points Total) The impulse response f of an LTI system is shown in the figure below:



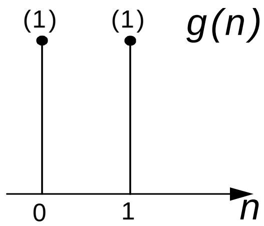
For time samples n not shown in the figure, $f(n) = 0$.

(a) (7 Points) Determine an expression for, and clearly sketch and label the salient features of, the magnitude $|F(\omega)|$ and phase $\angle F(\omega)$ of the frequency response of the LTI system over the frequency interval $-\pi \leq \omega < \pi$.

(b) (8 Points) Suppose the LTI system of part (a) is interconnected—according to the cascade structure shown below—with another LTI system having impulse response g.

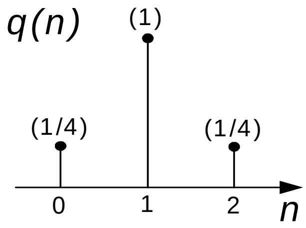


The impulse response g is shown below; $g(n) = 0$ for values of n not shown in the figure.



Determine, clearly sketch, and label the salient features of, h, the overall impulse response of the cascade interconnection of the LTI systems.

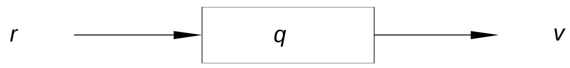
(c) (10 Points) Consider an LTI system whose impulse response q is as shown below; $q(n) = 0$ for values of n not shown in the figure.



Suppose an input signal

$$r(n) = \cos\left(\frac{7\pi n}{3} + \frac{\pi}{3}\right)$$

is applied to the LTI system q, as shown in the figure below.



Determine a simple expression for the corresponding output signal v.

Problem 6 (15 Points Total) Consider a continuous-time signal x whose value $x(t)$ at time t represents a voltage and has the unit of volt (V). The time t has the unit of second (sec).

(a) (4 Points) Suppose x is periodic and has the following Fourier series expansion:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}.$$

What is the unit of ω_0?

What is the unit of X_k?

(b) (6 Points) Suppose x has a continuous-time Fourier transform, in terms of which $x(t)$ can be represented as follows:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega.$$

What is the unit of ω?

What is the unit of dω?

What is the unit of $X(\omega)$?

Do X_k and $X(\omega)$ have the same unit?

(c) (5 Points) We now sample the continuous-time signal x once every T seconds, yielding the following discrete-time signal y:

$$y(n) = x(nT) \qquad \text{for all integers } n.$$

Let Y denote the discrete-time Fourier transform of the signal y. In terms of Y, y can be represented as follows:

$$y(n) = \frac{1}{2\pi} \int_0^{2\pi} Y(\Omega) e^{i\Omega n} d\Omega.$$

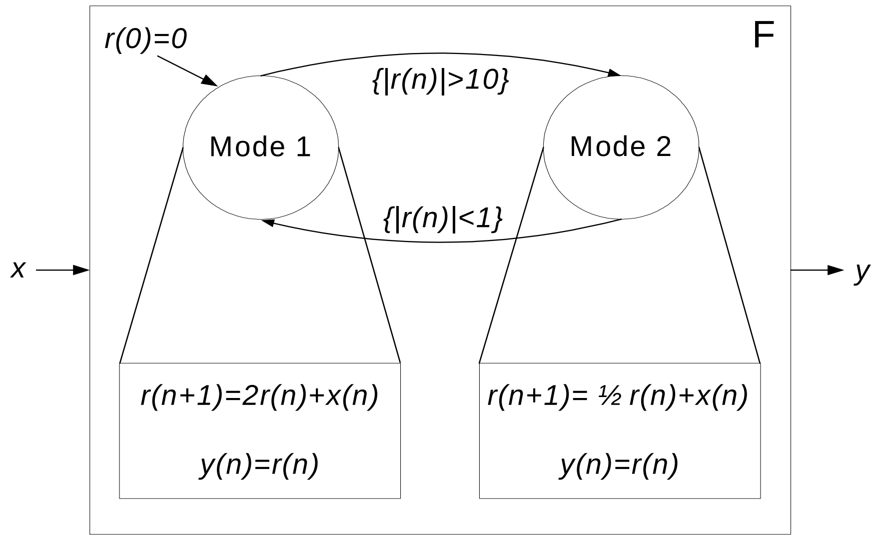
What is the unit of Ω?

What is the unit of dΩ?

What is the unit of $Y(\Omega)$?

Are the units of ω and Ω the same, or are they different? Explain why.

Problem 7 (20 Points Total) Consider the following discrete-time system F, with input x and output y:



At any time n, the system can be in one of two modes. Within each mode, the dynamics of the system are given by a linear state machine. The system initially starts in Mode 1 with $r(0) = 0$. It switches immediately to Mode 2 at the first instant n when $|r(n)| > 10$, such that the transition from time n to time $n + 1$ and thereafter is given by the dynamics of Mode 2. The system switches back to Mode 1 the first time when $|r(n)| < 1$, and so forth.

(a) (5 Points) Suppose the input x is an impulse (i.e., $x(n) = \delta(n)$)for all n. Plot the output y from time 0 to 20.

(b) (9 Points) The overall system F is a state machine. Does $r(n)$ constitute a state of the overall system? If so, give the state space, NextState function, and Output function for the state machine. If $r(n)$ is not a state, define an appropriate state for the overall system and the associated state space, NextState function, and Output function.

(c) (6 Points) Suppose we modify the system such that whenever it switches to Mode 2, it stays there only for three consecutive time instants before it switches back to Mode 1 (because it is too expensive to keep the system in Mode 2 for too long.) The condition for switching from Mode 1 to Mode 2 remains the same as before. Define an appropriate state for this new system and give the corresponding state space.

Problem 8 (10 Points Total) Let

$$x(t) = \cos(2\pi f_1 t) \qquad \text{for all } t$$

be a continuous-time sinusoid of frequency f_1 Hz. Suppose we sample the signal once every T seconds, i.e. the samples are $x(nT)$ for integers n.

(a) (5 Points) Sketch the signal x together with its samples, for $f_1 = 1$ and $T = 0.25$ second.

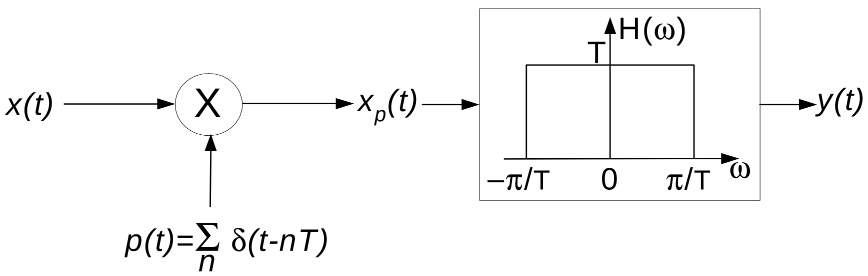
(b) (5 Points) Let $y(t) = \cos(2\pi f_2 t)$ for all t be a sinusoid of frequency f_2 Hz. For the sampling period $T = 0.25$ second, find the smallest frequency $f_2 > f_1$ such that y and x yield identical samples.

Problem 9 (15 Points Total) Consider a real, periodic, continuous-time signal x whose complex exponential Fourier series coefficients are given by:

$$X_k = \begin{cases} \frac{2}{3} & k = \pm 1 \\ \frac{1}{3} & k = \pm 2 \\ 0 & \text{otherwise} \end{cases},$$

The fundamental frequency of x is $\omega_0 = 10\pi$ radians/second (which is equivalent to $f_0 = 5\text{Hz}$).

The figure below depicts a system wherein the signal x is sampled by an infinite train of continuous-time impulses with periodicity T. The continuous-time signal x_p is subsequently processed by a real, continuous-time LTI filter whose frequency response H is that of the ideal low-pass filter shown in the figure.



(a) (5 Points) Give a simple expression for the input x in terms of one or more real sinusoids.

(b) (10 Points) If the sampling frequency is $f_s = 15\text{Hz}$ (corresponding to $\omega_s = 30\pi$ radians/second), find a simple time-domain expression for the continuous-time output $y(t)$, and interpret your answer in terms of what has happened to the various frequency components of the input x as a result of sampling at this particular frequency.

(Note: This should not involve any sophisticated or messy algebraic manipulation. Determine how spectrum of the input $x(t)$ looks graphically, infer the shape of the spectrum of the intermediate continuous-time signal x_p, and then determine what remains after the low-pass filtering by H. The time-domain expression for $y(t)$ is then forthcoming.).

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.