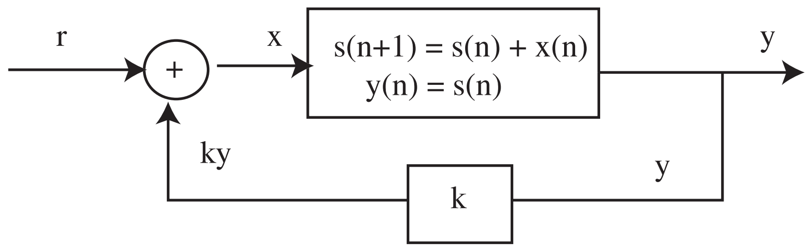
EECS 20. Midterm No. 2 Solution April 9, 2004.

1. **20 points** The block diagram of a feedback composition of a discrete-time system is given below:



The state s, input signal x and output signal y are related by the update equation:

$$\begin{aligned} s(n+1) &= s(n) + x(n) \\ y(n) &= s(n) \end{aligned}$$

(a) **6 points** Find the zero-state impulse response of this system.

Anwser The impulse response is

$$\forall n \geq 0, h(n) = \begin{cases} 0, & n = 0 \\ 1, & n \geq 1 \end{cases}$$

(b) **6 points** Find the update equation for the feedback system with input signal r, output signal y and state s.

Answer We have $x(n) = r(n) + ky(n) = x(n) + ks(n)$. So the update equation is:

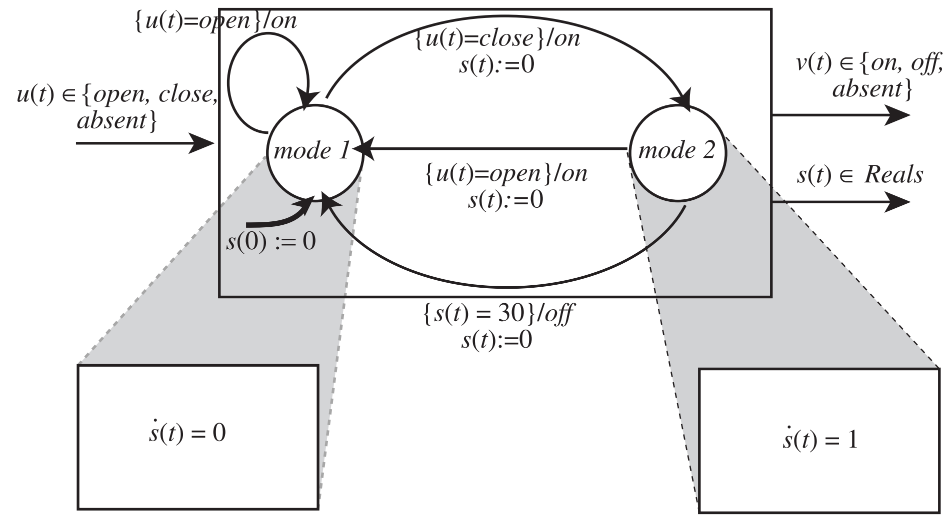
$$\begin{aligned} s(n+1) &= [1+k]s(n) + r(n) \\ y(n) &= s(n) \end{aligned}$$

(c) **8 points** Find the zero-state impulse response for the feedbac composition, when the 'gain' $k = -0.5$.

Answer The zero-state impulse response for the feedback composition is

$$\forall n \geq 0, h(n) = \begin{cases} 0, & n = 0 \\ (1+k)^{n-1} = (0.5)^{n-1}, & n \geq 1 \end{cases}$$

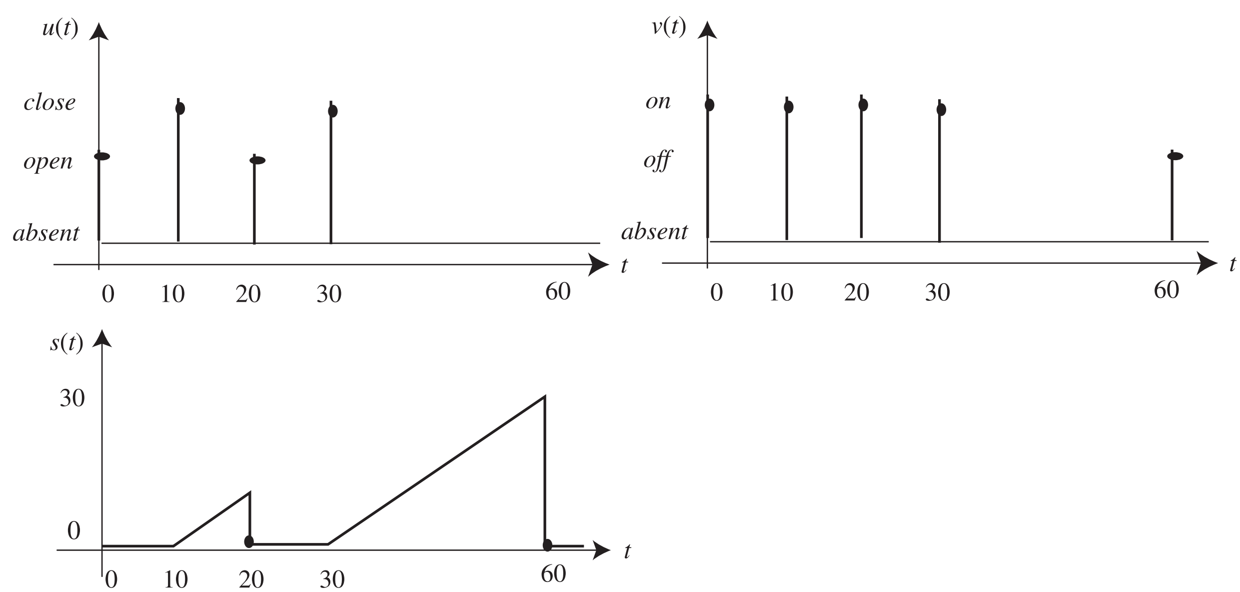
2. **20 points** The figure below is a partial hybrid system description of the dome light controller of an automobile.



When someone opens the door ($u(t) = open$), the light is turned on ($v(t) = on$). After the door is closed ($u(t) = close$) for 30 seconds, the light is turned off ($v(t) = off$). Note that the door must be closed for the entire 30 seconds, before the light is turned off.

(a) **10 points** Design the transitions (including guard, action, and output) so that the system meets this specification.

(b) **10 points** Plot the output signal $v(t)$ and the trajectory of the refinement state $s(t)$, $0 \leq t \leq 60$, when the input signal is as shown below.



3. **15 points** The continuous-time signal x is given by (t is in seconds)

$$\forall t \in R, \quad x(t) = \cos(2\pi \times 60 + \pi/4) + 2\cos(2\pi \times 120 + \pi/8) + 3\cos(2\pi \times 180 + \pi/12).$$

(a) **5 points** Is x periodic? If it is, what is its period?
Answer $\boxed{\text{Yes}}$, it is periodic. The period is $\boxed{\text{1/60 sec.}}$

(b) **10 points** The signal x is input to a LTI system whose frequency response is

$$\forall \omega \in R, \quad H(\omega) = \begin{cases} 1, & |\omega| < 2\pi \times 150, \\ 0.5, & \text{otherwise} \end{cases}$$

What is the output signal y? Is y periodic? If it is, what is its period?
Answer The output signal is

$$\boxed{\forall t, \quad y(t) = \cos(2\pi \times 60 + \pi/4) + 2\cos(2\pi \times 120 + \pi/8) + 1.5\cos(2\pi \times 180 + \pi/12)}$$

$\boxed{\text{Yes}}$, y is periodic. The period is $\boxed{\text{1/60 sec.}}$

4. **25 points** A LTI system with input signal x and output signal y is described by the differential equation

$$\frac{dy}{dt} + 0.5y(t) = x(t), \quad t \in R.$$

   (a) **10 points** Suppose the input signal is $\forall t, x(t) = e^{i\omega t}$, where ω is fixed. What is the output signal y?

   Answer The output signal is $\forall t, y(t) = H(\omega)e^{i\omega t}$. Substitution into the differential equation gives

$$i\omega H(\omega)e^{i\omega t} + 0.5H(\omega)e^{i\omega t} = e^{i\omega t},$$

   so

$$H(\omega) = \frac{1}{0.5 + i\omega}.$$

   Hence

$$\boxed{\forall t, \quad y(t) = \tfrac{1}{0.5+i\omega}e^{i\omega t}}$$

   (b) **5 points** What is the frequency response,

$$\forall \omega \in R, \quad H(\omega) =$$

   Answer

$$\boxed{H(\omega) = \tfrac{1}{0.5+i\omega}.}$$

   (c) **10 points** What is the magnitude and phase of the frequency response for $\omega = 0.5$ rad/sec?

$$|H(0.5)| =$$

$$\angle H(0.5) =$$

   Answer

$$|H(0.5)| = |\frac{1}{0.5 + i0.5}| = \boxed{\sqrt{2}}$$

$$\angle H(0.5) = \boxed{-\tfrac{\pi}{4}}$$

5. **20 points**

(a) **10 points** Consider a continuous-time system $S : [R \to R] \to [R \to R]$

i. Suppose

$$\forall x, \forall t, \quad S(x)(t) = x(t-2).$$

Is S time-invariant? Why?

Answer $\boxed{\text{Yes}}$, because the system is $S = D_2$ (delay by 2), so for all T, $D_2 \circ D_T = D_{2+T} = D_{T+2} = D_T \circ D_2$, and the system is time-invariant.

ii. Suppose

$$\forall x, \forall t, \quad S(x)(t) = x(2t).$$

Is S time-invariant? Why?

Answer $\boxed{\text{No}}$, because consider the signal $\forall t, x(t) = t$. Then $y(t) = S(x)(t) = 2t$, so

$$D_T \circ S(x)(t) = D_T(y)(t) = 2(t - T).$$

And $z(t) = D_T(x)(t) = t - T$, so

$$S \circ D_T(x)(t) = S(z)(t) = z(2t) = 2t - T.$$

So $D_T \circ S \neq S \circ D_T$.

(b) A discrete-time linear system produces output v when the input is the step u. What is the output h when the input is the impulse δ?