Please use these sheets for your answer and your work. Use the backs if necessary. Write clearly and put a box around your answer, and show your work.

Print your name and lab day and time below

Name: ________________________________
Lab time: _____________________________

Problem 1:
Problem 2:
Problem 3:
Problem 4:
Problem 5:
Total:
1. **20 points** The block diagram of a feedback composition of a discrete-time system is given below:

![Block Diagram]

The state $s$, input signal $x$, and output signal $y$ are related by the update equation:

\[
\begin{align*}
  s(n+1) &= s(n) + x(n) \\
  y(n) &= s(n)
\end{align*}
\]

(a) **6 points** Find the zero-state impulse response of this system.

(b) **6 points** Find the update equation for the feedback system with input signal $r$, output signal $y$ and state $s$.

(c) **8 points** Find the zero-state impulse response for the feedback composition, when the ‘gain’ $k = -0.5$
2. **20 points** The figure below is a partial hybrid system description of the dome light controller of an automobile.

![Hybrid System Diagram](image)

When someone opens the door \( u(t) = \text{open} \), the light is turned on \( v(t) = \text{on} \). After the door is closed \( u(t) = \text{close} \) for 30 seconds, the light is turned off \( v(t) = \text{off} \). Note that the door must be closed for the entire 30 seconds, before the light is turned off.

(a) **10 points** Design the transitions (including guard, action, and output) so that the system meets this specification.

(b) **10 points** Plot the output signal \( v(t) \) and the trajectory of the refinement state \( s(t) \), \( 0 \leq t \leq 60 \), when the input signal \( u \) is as shown below.
3. **15 points** The continuous-time signal $x$ is given by ($t$ is in seconds)

$$
\forall t \in \mathbb{R}, \quad x(t) = \cos(2\pi \times 60 + \pi/4) + 2 \cos(2\pi \times 120 + \pi/8) + 3 \cos(2\pi \times 180 + \pi/12).
$$

(a) **5 points** Is $x$ periodic? If it is, what is its period?

(b) **10 points** The signal $x$ is input to a LTI system whose frequency response is

$$
\forall \omega \in \mathbb{R}, \quad H(\omega) = \begin{cases} 
1, & |\omega| < 2\pi \times 150, \\
0.5, & \text{otherwise}
\end{cases}
$$

What is the output signal $y$? Is $y$ periodic? If it is, what is its period?
4. **25 points** A LTI system with input signal $x$ and output signal $y$ is described by the differential equation

$$\frac{dy}{dt} + 0.5y(t) = x(t), \quad t \in R.$$ 

(a) **10 points** Suppose the input signal is $\forall t, x(t) = e^{i\omega t}$, where $\omega$ is fixed. What is the output signal $y$?

(b) **5 points** What is the frequency response, 

$$\forall \omega \in R, \quad H(\omega) =$$

(c) **10 points** What is the magnitude and phase of the frequency response for $\omega = 0.5$ rad/sec? 

$$|H(0.5)| =$$

$$\angle H(0.5) =$$
5. 20 points

(a) 10 points Consider a continuous-time system $S : [R \rightarrow R] \rightarrow [R \rightarrow R]$ 

i. Suppose 
\[ \forall x, \forall t, \quad S(x)(t) = x(t - 2). \]
Is $S$ time-invariant? Why?

ii. Suppose 
\[ \forall x, \forall t, \quad S(x)(t) = x(2t). \]
Is $S$ time-invariant? Why?

(b) A discrete-time linear system produces output $v$ when the input is the step $u$. What is the output $h$ when the input is the impulse $\delta$?
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