EECS 20. Midterm No. 1
February 27, 2004.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: ______________________________

Lab time: ___________________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. **30 points. 5 points for each part.** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $\forall\, n \in \textit{Integers}, \quad (n, n+1, \sqrt{n}) \in \textit{Integers}^3$.

   (b) If A contains 6 elements, $P(A)$ contains 64 elements. $P(A)$ denotes the power set of A.

   (c) The function *square*: $[0, \infty) \to [0, \infty)$, given by $\forall x, f(x) = x^2$, is one-to-one and onto.

   (d) Consider a deterministic state machine with 5 states, $\textit{Inputs} = \{0, 1\}$, $\textit{Outputs} = \{0, 1\}$. Suppose the signal $y : \textit{Nats}_0 \to \{0, 1\}$ is the output response to the input signal $x = (0, 0, 0, ...)$ (all zeros). Then y is eventually periodic, i.e., there is an integer $1 \le p \le 5$ such that

   $$\forall n \ge 4, \quad y(n) = y(n+p).$$

   (e) Suppose state machine A simulates state machine B, A is deterministic and B is possibly non-deterministic. Then B simulates A.

   (f) Consider two state machines A and B with state spaces $\textit{States}_A$ and $\textit{States}_B$. If in each state machine, all states are reachable, then in the cascade composition, all states in $\textit{States}_A \times \textit{States}_B$ are reachable.

2. **10 points** The following Matlab program plots the graph of the function y.

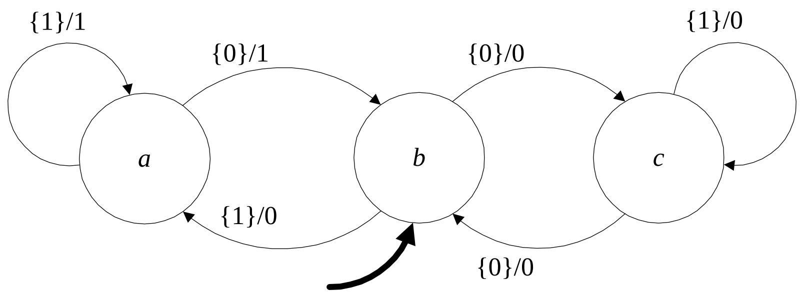
```
denseTime = [-1:0.01:1];
y = exp(-denseTime).*sin(10*pi*denseTime);
plot(denseTime,y);
```

(a) Domain of y =

(b) Range of y =

(c) Provide a mathematical expression:
$\forall x \in$ *Domain of y*, $\quad y(x) =$

3. **25 points. 2 points for parts (a)-(h), (j), 7 points for (i).** Consider the state transition diagram shown below.



(a) Add the transitions corresponding to the input symbol *absent* and give each of the following:

(b) *States* =

(c) *Inputs* =

(d) *Outputs* =

(e) *OutputSignals* =

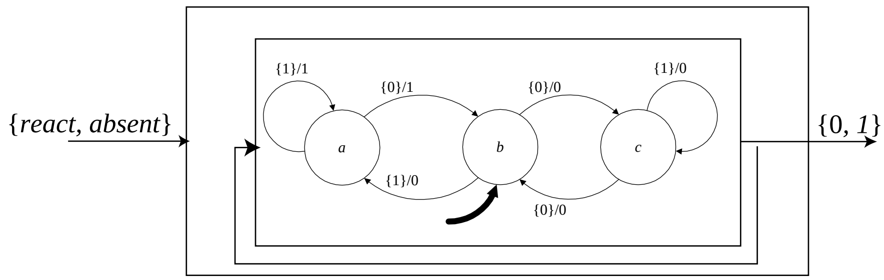
(f) Give the domain and range of the *update* function.

(g) Fill in the table for *update*:

current state	(*next state, output symbol*) under specified input symbol		
	1	0	*absent*
a			
b			
c			

(h) *initialState* =

(i) Compose this state machine in a feedback loop, where its output is connected to its input as in the figure below. Assume the output of the composition is the output of this state machine. Draw the state transition diagram for the composition, taking as its input alphabet the set $\{react, absent\}$.



(j) Which states can be reached from the initial state in the feedback machine?

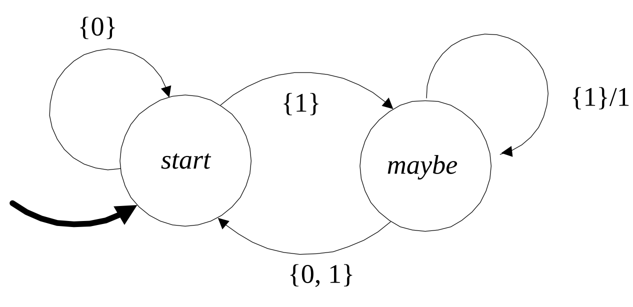
4. **15 points. 8 points for (a), 7 points for (b)**

(a) Design a state machine M, with *Inputs* $=$ *Outputs* $= \{0, 1, absent\}$, that has three states, and whose input-output function F is given by (neglecting the stuttering input *absent*):

$\forall x \in InputSignals, \forall n \in Nats_0$,

$$F(x)(n) = \begin{cases} 1, & \text{if } (x(n-2), x(n-1), x(n)) = (1,1,1) \\ absent, & \text{else} \end{cases}$$

(b) Now consider the non-deterministic machine N below.



Determine whether N simulates your machine M and write down the relevant simulation relation, if any.

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