EECS 20. Final Exam
May 17, 2004.

Please use these sheets for your answer. **Write clearly and show your work on the sheets in the back.** Please check that you have 10 numbered pages.

Print your name and lab time below

Name: ______________________________

Lab time: ___________________________

Problem 1 (10):

Problem 2 (15):

Problem 3 (10):

Problem 4 (15):

Problem 5 (15):

Problem 6 (15):

Problem 7 (20):

Total:

1. **10 points** A state machine has the same input and output alphabet, $\{0, 1, \textit{absent}\}$.

   (a) **3 points** Its inputs signals are:

   $$\textit{InputSignals} =$$

   Its output signals are:

   $$\textit{OutputSignals} =$$

   (b) **7 points** For any input signal x, the output signal y satisfies $y(n) = 0$ if $(x(0), ..., x(n))$ contains an *equal* number of 0's and 1's; and $y(n) = 1$, otherwise. Design a state machine (give its diagram and indicate the initial state) that has this input-ouput relationship.

2. **15 points** Consider the linear difference equation,

$$y(n) = x(n-2) + x(n-1) + x(n), \quad n \geq 0 \tag{1}$$

(a) **5 points** Give a $[A, b, c^T, d]$ representation of a state machine that satisfies this input-output relationship. What is the state $s(n)$ of your state machine in terms of x, y?

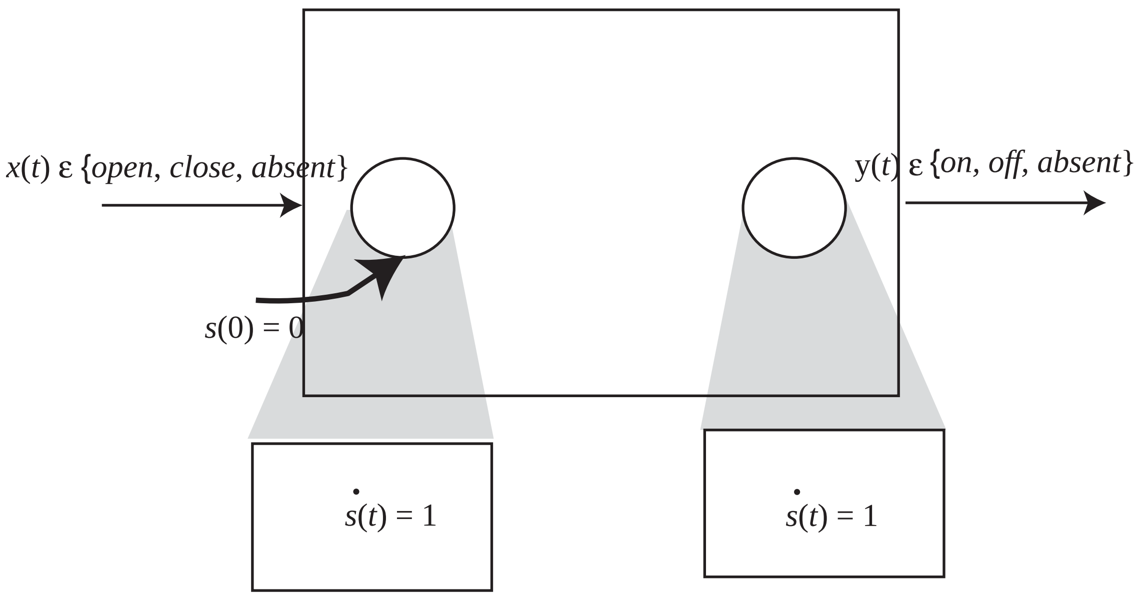
$s(n) =$

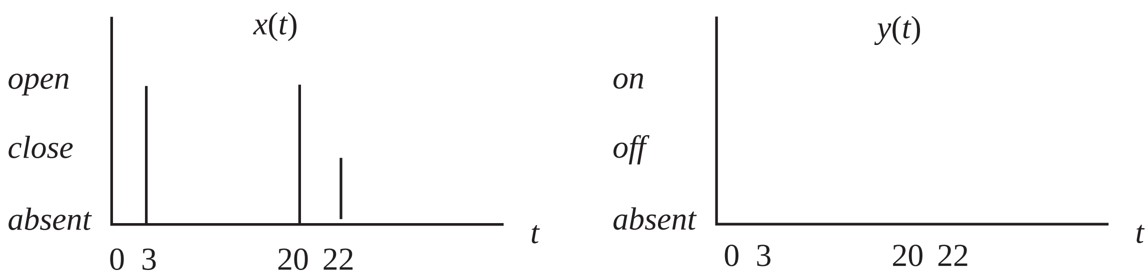
(b) **3 points** What is the zero-state impulse response of this system?

(c) **4 points** What is $y(n), n \geq 0$, if $x(-1) = x(-2) = 0$ and $x(n) = 1, n \geq 0$?

(d) **3 points** Design a tapped-delay line (give its signal flow graph) that implements (1).

3. **10 points** The figure below is an incomplete description of a controller. When someone presses the *open* button, the output is turned *on* and 15 sec later it is turned *off*. If the *open* button is pressed before the output is *off* the output stays *on* for 15 sec beyond the last time the *open* button was pressed. If someone presses *close* while the output is *on*, it is immediately turned *off*.





(a) **7 points** Design the guards, actions, and outputs for the transitions so as to meet this specification. Two modes are available, as shown in the diagram. However, you may use only one mode.

(b) **3 points** Sketch the output signal y when the input signal x is as shown. Mark all time instances t when y changes value.

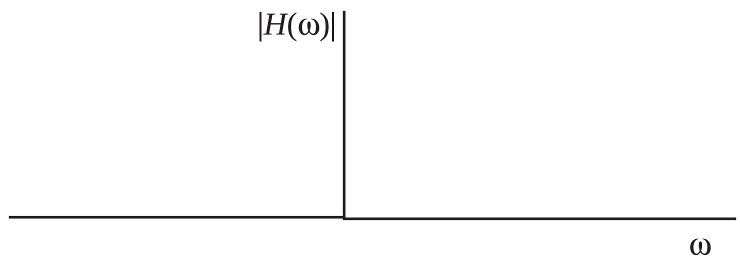
4. **15 points** A linear system with input x and output y is described by the second-order differential equation

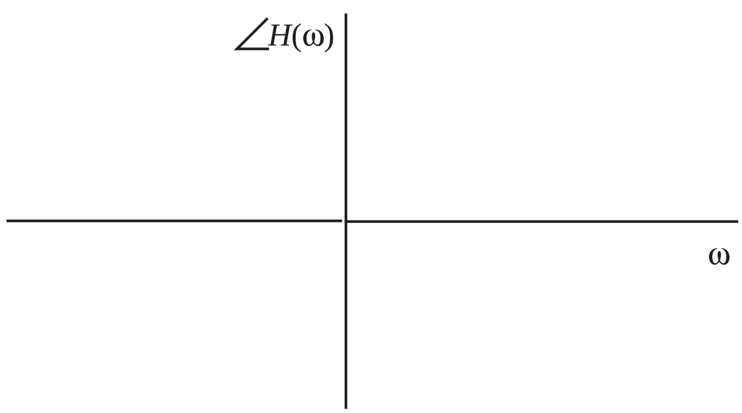
$$\ddot{y}(t) + 2\dot{y}(t) + y(t) = x(t).$$

(a) **8 points** Find the frequency response H of this system. Give simple expressions for the magnitude and phase responses: $\forall \omega$,

$$\begin{aligned} H(\omega) &= \\ |H(\omega)| &= \\ \angle H(\omega) &= \end{aligned}$$

(b) **7 points** Sketch the magnitude and phase response below. Carefully mark the values for $\omega = 0, \pm 1, \pm\infty$.





5. **15 points** Evaluate the convolution integral $y_i = h_i * x$ when $x : R \to R$ is the unit step, $x(t) = 0, t < 0; = 1, t \geq 0$, and $h_i : R \to R$ is as given below, $i = 1, 2, 3$.

   (a) **5 points** $h_1(t) = 0, t < 0; = e^{-t}, t \geq 0$.

   (b) **5 points** $h_2(t) = e^{t}, t < 0; = 0, t \geq 0$.

   (c) **5 points** $h_3(t) = e^{t}, t < 0; = e^{-t}, t \geq 0$.

6. **15 points** This problem concerns the various Fourier transforms.

(a) **3 points** The exponential Fourier series of the signal x,

$$\forall t \in R, \quad x(t) = \cos(2\pi t) + \sin(3\pi t),$$

is

(b) **5 points** The Fourier transform of the signal z,

$$\forall t \in R, \quad z(t) = e^{-t}, t \geq 0; = 0, t < 0,$$

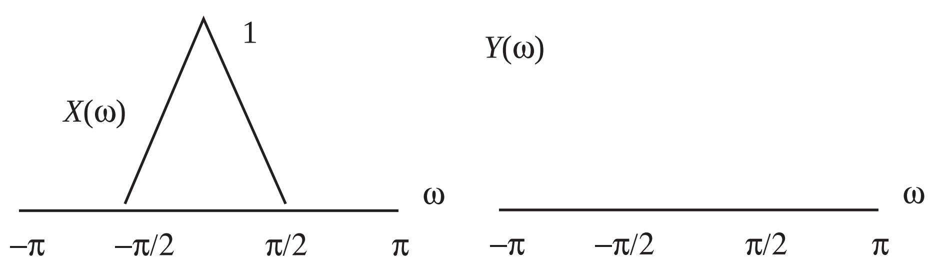
is

and the Fourier transform of the signal y,

$$\forall t \in R, \quad y(t) = z(t) e^{i\omega_0 t},$$

(in which z is as above) is

(c) **7 points** Suppose the DTFT X of a signal x : *Ints* $\to$ *Complex* is as shown below.



i. Prove that the signal x is real-valued.

ii. Suppose the signal y is constructed by: $y(k) = x(k/2)$, if k is even; and $y(k) = 0$, if k is odd. What is the DTFT Y of y in terms of X, and sketch Y above.

7. **20 points** In the figure on the next page, the left column shows three time signals, $x, p, y \in$ *ContSignals*.

   (a) **5 points** Write down expressions for the corresponsing Fourier Transforms X, P, Y.

$$X(\omega) \quad =$$

$$P(\omega) \quad =$$

$$Y(\omega) \quad =$$

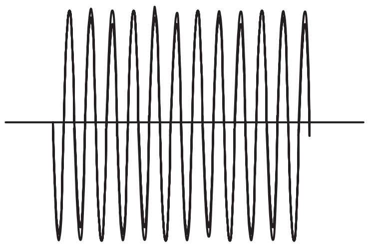
   (b) **5 points** Plot these Fourier Transforms in the column on the right. Mark the values at $\omega = 0$. Also, on the ω-axis, indicate the frequencies where the Fourier Transform is not zero.

   (c) **5 points** Suppose the signal y is sampled every 0.01 s, i.e. the sampling frequency is 100 Hz. The sampled signal is called $z \in$ *DiscSignals*. Write down an expression for the DTFT Z of z in terms of Y.

$$Z(\omega) =$$

   (d) **5 points** Sketch a plot of Z in the figure.

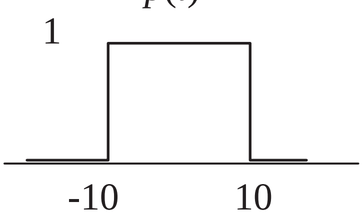
$x(t) = \cos(20\pi t)$

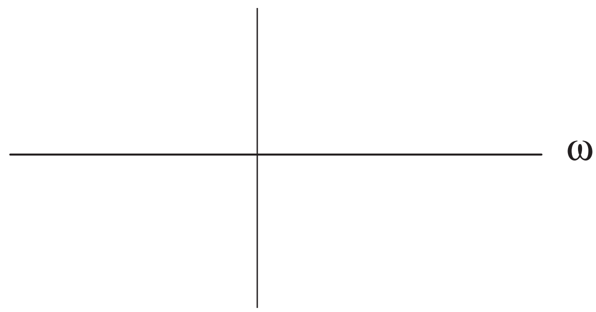


$X(\omega)$

$P(\omega)$

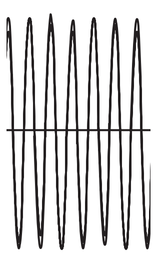
$p(t)$

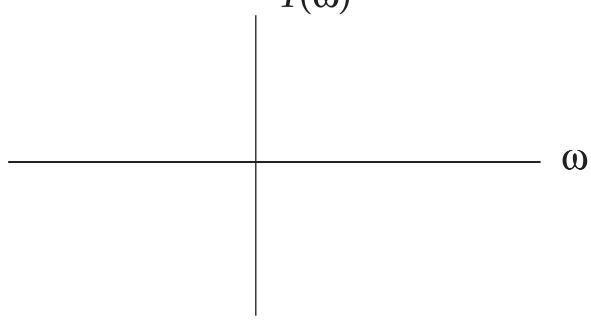




$Y(\omega)$

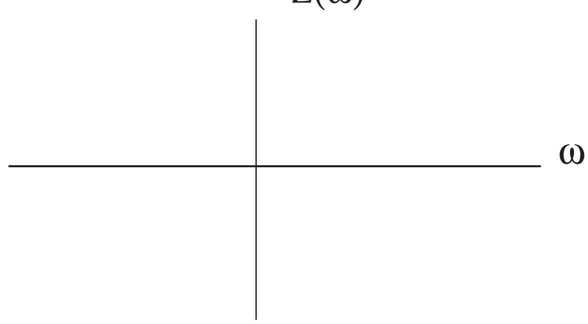
$y(t) = x(t)p(t)$





$Z(\omega)$

$z(k) = y(0.01k)$



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