EECS 20. Midterm No. 1
February 28, 2003.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: ____________________________________

Lab time: _________________________________

Problem 1:

Problem 2:

Problem 3:

Total:

1. **50 points. 5 points for each part.** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $\forall\, n \in \textit{Integers}, \quad (n, n+1, n^2) \in \textit{Integers}^3$.

   (b) $\forall\, x \in \textit{Integers}, \quad (x, x+1) \in \{1, 2, 3\}^2$.

   (c) If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $\exists\, x \in A$ such that $\forall y \in B$, $x \geq y$.

   (d) If A contains 5 elements, $P(A)$ contains 32 elements, where $P(A)$ denotes the power set of A.

   (e) Let A, B be any sets. Let $identity\colon A \to A$ be the identity function, that is, $\forall x \in A, f(x) = x$. Let $g\colon A \to B$ be any function. Then $g \circ f = g$.

   (f) Let $f\colon \textit{Reals} \to [-1, 1]$ be the function $\forall x \in \textit{Reals}$, $f(x) = \sin(x)$. Then f is onto.

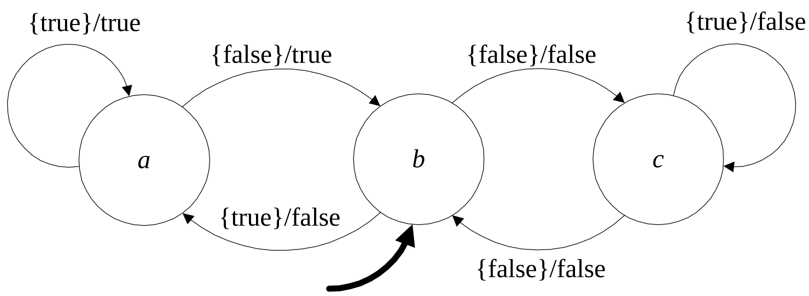
(g) Let $cube: \textit{Reals} \to \textit{Reals}$ be the function $\forall x \in \textit{Reals}$, $cube(x) = x^3$. Then $cube$ is one-to-one and onto.

(h) Consider the signal spaces $\textit{Signals}_1 = [\textit{Naturals}_0 \to \{true, false\}]$, and $\textit{Signals}_2 = [\textit{Naturals}_0 \to \{true, false, maybe\}]$. Then $\textit{Signals}_1 \subset \textit{Signals}_2$.

(i) Given two state machines A and B, if A simulates B and A is deterministic, then B simulates A.

(j) Consider two state machines A and B with state spaces $\textit{States}_A$ and $\textit{States}_B$. If in each state machine, all states are reachable, then in the cascade composition, all states in $\textit{States}_A \times \textit{States}_B$ are reachable.

2. **35 points. 3 points for parts (a)-(g), 14 points for (h).** Consider the state transition diagram shown below.



Give each of the following:

(a) *States* =

(b) *Inputs* =

(c) *Outputs* =

(d) *OutputSignals* =

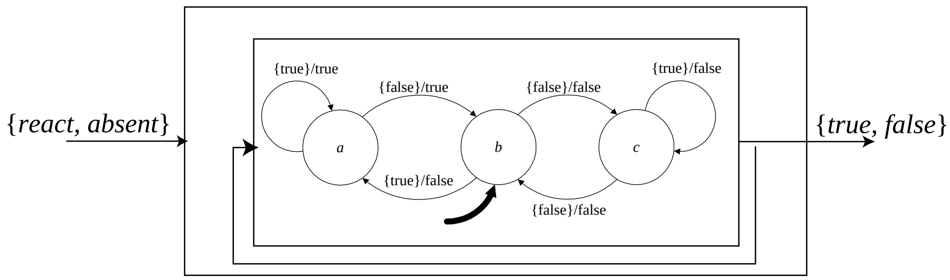
(e) Give the domain and range of the *update* function.

(f) Fill in the table for *update*:

current state	(*next state, output symbol*) under specified input symbol		
	true	false	*absent*
a			
b			
c			

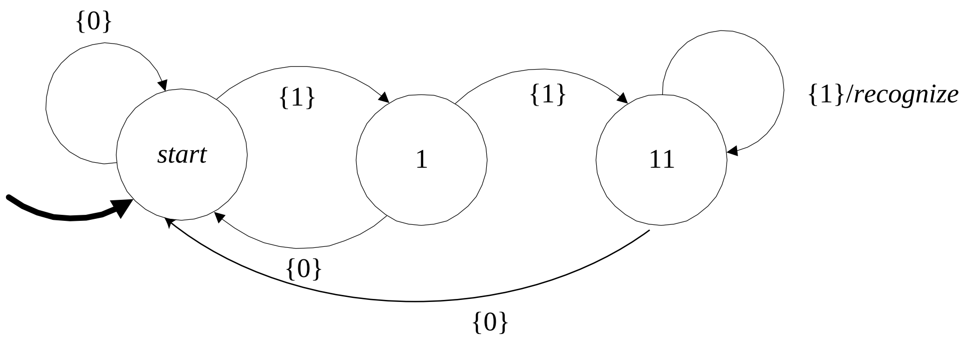
(g) *initialState* =

(h) Compose this state machine in a feedback loop, where its output is connected to its input as in the figure below. Assume the output of the composition is the output of this state machine. Draw the state transition diagram for the composition, taking as its input alphabet the set $\{react, absent\}$.



3. **15 points. 5 points for (a), 10 points for (b)**

(a) Consider the deterministic machine A. It is similar to the machine *CodeRecognizer* studied in the text and in the homework.

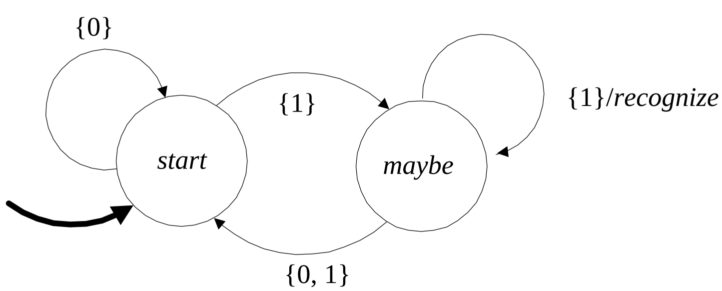


Let x denote an input signal and y the corresponding output signal. Complete the expression for $y(n)$ below, ignoring stuttering inputs, (i.e. replace the $\cdots$ by an expression involving x)

$$\forall x \in \textit{InputSignals}, \qquad \forall n \in \textit{Naturals}_0,$$

$$y(n) = \begin{cases} \textit{recognize}, & \text{if } \cdots \\ \textit{absent}, & \text{otherwise} \end{cases}$$

(b) Now consider the non-deterministic machine B.



Determine whether B simulates A and write down the relevant simulation relation, if any.

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