EECS 20. Midterm No. 2
April 16, 2002.

Please use these sheets for your answer and your work. Use the backs if necessary. Write clearly and put a box around your answer, and show your work.

Print your name and lab TA’s name below

Name: ________________________________
Lab TA: ______________________________

Problem 1:
Problem 2:
Problem 3:
Problem 4:
Total:
1. **30 points.** Consider a continuous-time signal \( x : \text{Reals} \to \text{Reals} \) defined by

\[
\forall t \in \text{Reals}, \quad x(t) = 3 + 2 \sin(3t) + 3 \cos(4t).
\]

(a) Obtain the Fourier series coefficients of \( x(t) \), i.e., find the coefficients \( A_0, A_1, A_2, \ldots \) and \( \phi_1, \phi_2, \ldots \) and \( w_0 \) such that

\[
x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(kw_0 t + \phi_k).
\]

(b) Obtain the Fourier series expansion for \( x(t) \), i.e., find the coefficients \( X_k \) for all \( k \in \text{Integers} \) such that

\[
x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ikw_0 t}.
\]
(c) Consider a continuous-time LTI system $\text{Filter} : [\text{Reals} \to \text{Reals}] \to [\text{Reals} \to \text{Reals}]$ with the following frequency response:

\[
\begin{align*}
H(w) &= 2, \quad |w| \geq 2, \\
H(w) &= 0, \quad |w| < 2.
\end{align*}
\]

Such a filter is an amplifying high-pass filter. Give a simple expression for the output $y(t)$ of the system, where $y = \text{Filter}(x)$. 

2. **20 points.** Consider a system whose input and output are related by
\[
\forall n \in \text{Integers}, \ y(n) = 2 \ x(n - 2) + 1.1 \ y(n - 1).
\]

(a) Construct a state-space model for the system. It is sufficient to give the state definition, the \(A\) matrix, vectors \(b\) and \(c\), and scalar \(d\).

(b) Give an expression for the zero-state impulse response.

(c) Recall that a system is stable if a bounded input always produces a bounded output. Is this system stable? Explain.
3. **25 points.** Consider discrete-time systems with input $x : \text{Integers} \rightarrow \text{Reals}$ and output $y : \text{Integers} \rightarrow \text{Reals}$. Each of the following defines such a system. For each of the following, indicate whether it is linear only (L), time-invariant only (TI), both (LTI), or neither (N). Note that no partial credit will be given for these questions.

(a) $\forall n \in \text{Integers}, \quad y(n) = x^3(n - 10) = (x(n - 10))^3$

(b) $\forall n \in \text{Integers}, \quad y(n) = \cos(x(n))$

(c) $\forall n \in \text{Integers}, \quad y(n) = n$

(d) $\forall n \in \text{Integers}, \quad y(n) = \max\{|x(n)|, |x(n - 1)|\}$

(e) $\forall n \in \text{Integers}, \quad y(n) = x(-n)$
4. **15 points.** Consider a continuous-time LTI system $S$. Suppose that when the input is given by
\[
x(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}
\]
then the output $y = S(x)$ is given by
\[
y(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ \sin(\pi(t - 2)) & 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}
\]
for all $t \in \text{Reals}$.

(a) Carefully sketch these two signals.

(b) Give a simple expression and a sketch for the output of the same system if the input is
\[
x(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ -\sin(\pi(t - 1)) & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}
\]
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