EECS20, Spring 2002 – Solutions to Midterm 1

1. 15 points
   (a) Not well-formed: The feedback composition has more than one fixed point solution when the initial state of $B$ is 1 and of $C$ is $x$.
   (b) Well-formed: The feedback composition has a unique non-stuttering input for all reachable states.
   (c) Not well-formed: The output of $C$ is not a subset of the inputs of $A$.

2. 20 points
   (a) (7 points) State transition diagram for $A$ is shown below:

   ![State Transition Diagram](image)

   (b) (5 points) The following machine is bisimilar to $A$:
   
   $\text{initialState}=a$
   $\text{Inputs} = \{0,1,\text{reset},\text{absent}\}$
   $\text{Outputs} = \{0,1,\text{absent}\}$
   $\text{States}=\{a',c',d'\}$
(c) (3 points) \{ (a,a'), (b,a'), (c,c'), (d,d') \}

(d) (5 points)

3. **20 points**

(a) Yes, the machine $B$ can be defined via a finite state machine model. The set of Inputs, Outputs and States, and initialState are the same as in $A$; only the update function changes appropriately. (Note: It is possible to concoct examples where $B$ stays in its initial state for all possible inputs; i.e., all other states are unreachable. Nevertheless, $B$ still satisfies the properties of a FSM.)

(b) No. Consider the following counterexample:
In the arc-reversed machine, there are two arcs emanating from state $b$ for the same input.

4. **10 points**

In order for the system to be well-formed, it must have a unique non-stuttering fixed point. Using the input-output relationship

$$y(n) = cs(n) + dx(n)$$

and the feedback law $x(n) = ky(n)$, we obtain

$$y(n) = cs(n) + dk y(n)$$

$$y(n) = \frac{cs(n)}{1 - dk}.$$ 

This leads to the requirement that $1 - dk \neq 0$, or equivalently, $dk \neq 1$. 
