Problem 1:
Problem 2:
Problem 3:
Problem 4:
Total:
1. 15 points. Consider state machines A, B and C, described below by their state space, input alphabet, output alphabet, and state transition diagram.

Let

\[\text{Set1} = \{T, F, \text{absent}\}\]

\[\text{Set2} = \{1, 0, \text{absent}\}\]

State machine A:

\text{Inputs} = \text{Set2}, \text{Outputs} = \text{Set1}, \text{States} = \{a, b\}

State machine B:

\text{Inputs} = \text{Set1}, \text{Outputs} = \text{Set2}, \text{States} = \{1, 2\}

State machine C:

\text{Inputs} = \text{Set2}, \text{Outputs} = \text{Set1}, \text{States} = \{x, y\}
Classify whether these compositions are well-formed. Explain your answer.

(a) *Well-formed or not well-formed*

(b) *Well-formed or not well-formed*

(c) *Well-formed or not well-formed*
2. **20 points.** Consider the following state machine $A$:

Inputs = \{0,1,reset,absent\}

Outputs = \{0,1,absent\}

States = \{a,b,c,d\}

initialState = $a$

update function is given by the following table of values of (nextState, output symbol) under specified input symbol

<table>
<thead>
<tr>
<th>current state</th>
<th>0</th>
<th>1</th>
<th>reset</th>
<th>absent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(b,0)</td>
<td>(c,0)</td>
<td>(a,0)</td>
<td>(a,absent)</td>
</tr>
<tr>
<td>b</td>
<td>(a,0)</td>
<td>(c,0)</td>
<td>(a,0)</td>
<td>(b,absent)</td>
</tr>
<tr>
<td>c</td>
<td>(b,0)</td>
<td>(d,0)</td>
<td>(a,0)</td>
<td>(c,absent)</td>
</tr>
<tr>
<td>d</td>
<td>(d,0)</td>
<td>(a,1)</td>
<td>(a,0)</td>
<td>(d,absent)</td>
</tr>
</tbody>
</table>

(a) Draw the state transition diagram for state machine $A$.  

(b) Construct a simpler state machine which is bisimilar to \( A \). Give Inputs, Outputs, States, initialState, and a state transition diagram for this state machine.

(c) What is the bisimulation relation between the state machines in part (a) and (b)?

(d) Provide the state transition diagram of a one-state non-deterministic state machine which simulates \( A \).
3. **20 points.** Consider a general finite state machine $A$ with a certain set of *Inputs, Outputs, States, initial State* and state transition diagram. Now construct a new FSM $B$ with the same set of *Inputs, Outputs, States* and *initial State*, but with the arrows in the state transition diagram reversed; i.e., the direction of every arc in $B$ is the reverse of the corresponding arc in $A$. The figure below illustrates this for one specific arc of $A$ and the corresponding arc of $B$.

(a) Does the arc-reversed machine $B$ always make sense? That is, can we always define it via a finite state machine model? Explain.

(b) Is $B$ always deterministic if $A$ is? Prove or give a counterexample.
4. **10 points.** Consider a discrete-time system with one input port, one output port and one state. The current input, current output, and current state are denoted by \( x(n), y(n), \) and \( s(n), \) respectively. The equations describing this system are:

\[
\begin{align*}
    s(n+1) &= as(n) + bx(n) \\
    y(n) &= cs(n) + dx(n)
\end{align*}
\]

Furthermore, the output is multiplied by a feedback gain factor \( k \) and fed back as input.

\[
x(n) = k y(n).
\]

Derive conditions on the values of \( a, b, c, d, k \) under which the system is well-formed.
Use this page for overflow