Carefully read the questions. Use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. Write clearly and put a box around your answer, and show your work.

Print your name below

Last Name ___________________________ First ______________________
Name of your Lab TA ________________________________

Problem 1:
Problem 2:
Problem 3:
Problem 4:
Total:
1. **20 points** Let $x : \text{Reals} \rightarrow \text{Comps}$ be a continuous-time signal with Fourier Transform $X$. The **bandwidth** of $x$ is defined to be the smallest number $\Omega_x$ (rads/sec) such that $|X(\omega)| = 0$ for $|\omega| > \Omega_x$. If there is no such finite number, we say that the signal is not bandlimited and let $\Omega_x = \infty$.

Answer the following and give a brief justification for your answer.

(a) If $\forall t, x(t) = 1$, what is $X$ and what is the bandwidth of $x$?

(b) If $\forall t, x(t) = \delta(t)$ (Dirac delta), what is $X$ and what is the bandwidth of $x$?

(c) If $\forall t, x(t) = \cos(t)$, what is $X$ and what is the bandwidth of $x$?

(d) If $x$ has bandwidth $\Omega_x$ what is the bandwidth of the signal $2x$?

(e) If $x$ has bandwidth $\Omega_x$ and $y$ has bandwidth $\Omega_y > \Omega_x$, what is the bandwidth of the signal $x + y$?
2. \textbf{30 points} Suppose \( x \) is a continuous-time signal, with Fourier Transform \( X \).

(a) What are the units of \( \omega \) in \( X(\omega) \)?
(b) Write down the definition of \( y = \text{Sampler}_T(x) \).
(c) Let \( Y(\hat{\omega}) \) be the DTFT of \( y \). What are the units of \( \hat{\omega} \)?
(d) What is \( Y \) in terms of \( X \)?
(e) Suppose \( X \) is as shown in Figure 1. For what values of \( T \) will there be no aliasing?
(f) Sketch \( Y(\hat{\omega}) \) when \( T = 1/2 \) and when \( T = 3/4 \)?

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{\( X \) for problem 2}
\end{figure}
3. **30 points** Let \( x : \text{Ints} \to \text{Reals} \) be a discrete-time signal with DTFT \( X \). Let \( h : \text{Ints} \to \text{Reals} \) be another discrete-time signal with DTFT \( H \). Let \( y = h \ast x \), the convolution sum of \( h \) and \( x \).

(a) Give an explicit expression for \( y \) in terms of \( h \) and \( x \).

(b) Let \( X, H, Y \) be the DTFT of \( x, h, \) and \( y \), respectively. Express \( Y \) in terms of \( X, H \).

(c) Suppose

\[
X(\omega) = \begin{cases} 
1, & |\omega| \leq \pi/4 \\
0, & \pi/4 < |\omega| \leq \pi 
\end{cases}
\]

Find the signal \( x \).

(d) Suppose

\[
H(\omega) = \begin{cases} 
0, & |\omega| \leq \pi/4 \\
1, & \pi/4 < |\omega| \leq \pi 
\end{cases}
\]

Find \( y = h \ast x \)?
4. **20 points** Construct a state machine with

\[ \text{Inputs} = \{0, 1\}, \quad \text{Outputs} = \{\text{Yes, No, absent}\}, \]

such that for any input signal, the machine outputs *Yes* if the most recent three input values are 111, outputs *No* if the most recent three input values are 000, and in all other cases it outputs *absent*. In other words, if the input signal is

\[ u(0), u(1), \cdots, \]

then the output signal

\[ y(0), y(1), \cdots, \]

where

\[
y(n) = \begin{cases} 
\text{yes,} & \text{if } (u(n - 2), u(n - 1), u(n)) = 111 \\
\text{no,} & \text{if } (u(n - 2), u(n - 1), u(n)) = 000 \\
\text{absent,} & \text{otherwise}
\end{cases}
\]