EECS 20. Midterm 2. November 6, 1998

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. Write clearly and put a box around your answer. *You will lose points for sloppy work!*

Print your name below

Last Name________________ First_________________

Score:

Problem 1:_____

Problem 2:_____

Problem 3:_____

Problem 4:_____

Total: __________

1) **24 points**. Consider a continuous-time signal x with the following finite Fourier series expansion: for all $t \in$ *Reals*,

$$x(t) = \sum_{k=0}^{4} \cos(k\omega_0 t)$$

where $\omega_0 = \pi/4$ radians/second. Define $Sampler_T$: *ContSignals* → *DiscSignals* to be a sampler with sampling interval T (in seconds). Define *IdealDiscToCont* : *DiscSignals* → *ContSignals* to be an ideal bandlimited interpolation system. I.e., given a discrete-time signal $y(n)$, it constructs the continuous-time signal w where for all $t \in$ *Reals*,

$$w(t) = \sum_{k=-\infty}^{\infty} y(nT)\, p(t - nT)$$

where the pulse p is the sinc function,

$$p(t) = \frac{\sin(\pi t / T)}{\pi t / T}$$

a) Give an upper bound on T (in seconds) such that x = *IdealDiscToCont* ($Sampler_T$ (x)).
b) Suppose that T = 4 seconds. Give a *simple* expression for $y = Sampler_T(x)$.
c) For the same T = 4 seconds, give a *simple* expression for w = *IdealDiscToCont* ($Sampler_T(x)$).

2) **24 points**. Consider an LTI discrete-time system *Filter* with impulse response *h* where for all $n \in$ *Ints*,

$$h(n) = \sum_{k=0}^{7} \delta(n-k)$$

where δ is the Kronecker delta function.

a) Sketch *h*.
b) Suppose the input signal x : *Ints* $\rightarrow$ *Reals* is such that for all $n \in$ *Ints*, $x(n) = \cos(\omega n)$, where $\omega = \pi/4$ radians/sample. Give a *simple* expression for y = *Filter*(x).
c) Give the value for $H(\omega)$ for $\omega = \pi/4$ radians/sample, where H = *DTFT*(h).

3) **32 points**. Suppose that the frequency response *H* of a discrete-time LTI system *Filter* is given by: for all $\omega \in$ *Reals*,

$$H(\omega) = \cos(2\omega)$$

where ω has units of radians/sample. Give simple expressions for the output *y* when the input signal $x : Ints \to Reals$ is such that for all $n \in$ *Ints*, is each of the following is true:

a) $x(n) = \begin{cases} +1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$

b) $x(n) = 5$

c) $x(n) = \cos(\pi n/2)$

d) $x(n) = \cos(\pi n/4)$

4) **20 points** Let u be a discrete-time signal given by: for all $n \in$ *Ints*,

$$u(n) = \begin{cases} 1 & 0 \le n \\ 0 & \text{otherwise} \end{cases}.$$

This is called the **unit step** signal. Suppose that a discrete-time system H that is known to be LTI is such that if the input is u, the output is $y = H(u)$ given by: for all $n \in$ *Ints*,

$$y(n) = n\, u(n).$$

a) Find a simple expression for the output $w = H(p)$ when the input is p given by: for all $n \in$ *Ints*,

$$p(n) = \begin{cases} 2 & 0 \le n < 8 \\ 0 & \text{otherwise} \end{cases}.$$

Sketch w.

b) Find a simple expression for the impulse response h of H. Give a sketch of h.

4. Suppose you are given the following building blocks:

$$H(\omega) = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{otherwise} \end{cases}$$