Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. Write clearly and put a box around your answer. You will lose points for sloppy work!

Print your name below

Last Name__________________ First__________________

Score:
Problem 1:_____
Problem 2:_____
Problem 3:_____
Problem 4:_____

Total:________
1) **24 points.** Consider a continuous-time signal $x$ with the following finite Fourier series expansion: for all $t \in \mathbb{Reals}$,

$$x(t) = \sum_{k=0}^{4} \cos(k\omega_0 t)$$

where $\omega_0 = \pi/4$ radians/second. Define $\text{Sampler}_T : \text{ContSignals} \to \text{DiscSignals}$ to be a sampler with sampling interval $T$ (in seconds). Define $\text{IdealDiscToCont} : \text{DiscSignals} \to \text{ContSignals}$ to be an ideal bandlimited interpolation system. I.e., given a discrete-time signal $y(n)$, it constructs the continuous-time signal $w$ where for all $t \in \mathbb{Reals}$,

$$w(t) = \sum_{k=-\infty}^{\infty} y(nT)p(t - nT)$$

where the pulse $p$ is the sinc function,

$$p(t) = \frac{\sin(\pi t / T)}{\pi t / T}$$

a) Give an upper bound on $T$ (in seconds) such that $x = \text{IdealDiscToCont} (\text{Sampler}_T(x))$.

b) Suppose that $T = 4$ seconds. Give a simple expression for $y = \text{Sampler}_T(x)$.

c) For the same $T = 4$ seconds, give a simple expression for $w = \text{IdealDiscToCont} (\text{Sampler}_T(x))$. 
2) 24 points. Consider an LTI discrete-time system \textit{Filter} with impulse response \( h \) where for all \( n \in \text{Ints} \),

\[
h(n) = \sum_{k=0}^{7} \delta(n - k)
\]

where \( \delta \) is the Kronecker delta function.

a) Sketch \( h \).

b) Suppose the input signal \( x : \text{Ints} \to \text{Reals} \) is such that for all \( n \in \text{Ints} \), \( x(n) = \cos(\omega n) \), where \( \omega = \pi/4 \) radians/sample. Give a \textit{simple} expression for

\( y = \text{Filter}(x) \).

c) Give the value for \( H(\omega) \) for \( \omega = \pi/4 \) radians/sample, where \( H = DTFT(h) \).
3) **32 points.** Suppose that the frequency response $H$ of a discrete-time LTI system $Filter$ is given by: for all $\omega \in \text{Reals}$,

$$H(\omega) = \cos(2\omega)$$

where $\omega$ has units of radians/sample. Give simple expressions for the output $y$ when the input signal $x : \text{Ints} \to \text{Reals}$ is such that for all $n \in \text{Ints}$, is each of the following is true:

a) $x(n) = \begin{cases} +1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$

b) $x(n) = 5$

c) $x(n) = \cos(\pi n/2)$

d) $x(n) = \cos(\pi n/4)$
4) 20 points Let $u$ be a discrete-time signal given by: for all $n \in \text{Ints}$,

$$u(n) = \begin{cases} 1 & 0 \leq n \\ 0 & \text{otherwise} \end{cases}.$$ 

This is called the unit step signal. Suppose that a discrete-time system $H$ that is known to be LTI is such that if the input is $u$, the output is $y = H(u)$ given by: for all $n \in \text{Ints}$,

$$y(n) = n u(n).$$

a) Find a simple expression for the output $w = H(p)$ when the input is $p$ given by: for all $n \in \text{Ints}$,

$$p(n) = \begin{cases} 2 & 0 \leq n < 8 \\ 0 & \text{otherwise} \end{cases}.$$ 

Sketch $w$.

b) Find a simple expression for the impulse response $h$ of $H$. Give a sketch of $h$. 
4. Suppose you are given the following building blocks:

\[ H(\omega) = \begin{cases} 
1 & -W < \omega < W \\
0 & \text{otherwise}
\end{cases} \]