EECS 20. Final Exam 9 December 1998

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer**

Print your name below

Last Name ______________ First ______________

Problem 1	Problem 5
Problem 2	Problem 6
Problem 3	Problem 7
Problem 4	Total

1. **15 points**
Give a brief justification of your answer in each case.

(a) Find the smallest positive integer n such that

$$\sum_{k=0}^{n} \exp(2k \times 5\pi/12) = 0.$$

(b) Find $\theta \in [0, \pi/2]$ so that

$$Re[(1+i)\exp i\theta] = 0$$

(c) Find $A \in Comps$ so that

$$\forall t \in Reals, \quad A\exp(i\omega t) + A^* \exp(-i\omega t) = \cos(\omega_0 t + \pi/4)$$

where A^* is the complex conjugate of A.

2. **15 points**

Draw the following sets:

(a) $\{(x,y) \mid x^2 + y^2 = 1\}$

(b) $\{(x,y) \mid |x| + |y| = 1\}$

(c) $\{(x,y) \mid \max\{|x|, |y|\} = 1\}$

3. **15 points**

A relation F between a set X to a set Y is a subset $F \subset X \times Y$. Figure 1 shows a

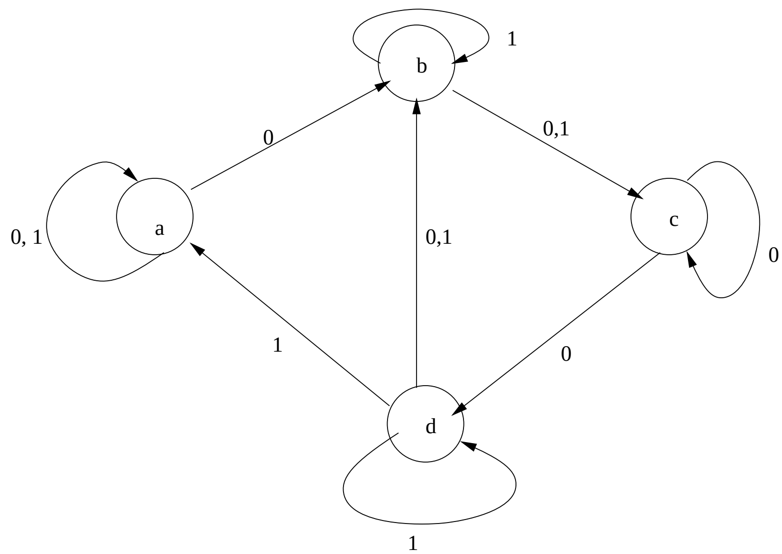

Figure 1: A graph. The edges are directional.

graph consisting of four nodes $X = \{a, b, c, d\}$. There are directional edges going from one node to another carrying labels 0 or 1 or both. Construct the following relations on $X \times X$

$$
\begin{aligned}
F_0 &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0\} \\
F_1 &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 1\} \\
F_{01} &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0 \text{ and an edge labeled } 1\} \\
F_{0or1} &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0 \text{ or} 1\}
\end{aligned}
$$

(a) Write each of these four relations as a list.

(b) Are the following assertions true or false?

$$F_{01} = F_0 \cap F_1, \; F_{0or1} = F_0 \cup F_1$$

(c) Define the relation

$$F_{00} = \{(x, y) \mid \text{ two consecutive edges labeled } 0 \text{ connect } x \text{ to } y\}$$

List F_{00}.

Write your answer on the next page.

Answer sheet for question 3.

4. **15 points** Consider a continuous-time LTI system H. Suppose that when the input x is given by

$$\forall t \in Reals, \quad x(t) = \begin{cases} 1, & \text{if } 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$

then the output y is given by

$$\forall t \in Reals, \quad y(t) = \begin{cases} 1, & \text{if } 0 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

Give an expression and a sketch for the output of the same system if the input is

$$\forall t \in Reals, \quad x'(t) = \begin{cases} 1, & \text{if } 0 \le t < 1 \\ -1, & \text{if } 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

5. **20 points** Suppose that the frequency response H of a discrete-time LTI system *Filter* is given by:

$$\forall \omega \in Reals, \quad H(\omega) = |\omega|.$$

where ω has units of radians/sample. Give simple expressions for the output y when the input signal $x : Ints \to Reals$, where $Ints = \{\cdots -2, -1, 0, 1, 2, \cdots\}$, is such that $\forall n \in Ints$ each of the following is true:

(a) $x(n) = cos(\pi n/2)$.

(b) $x(n) = 5$.

(c) $x(n) = \begin{cases} +1, & n \text{ even} \\ -1, & n \text{ odd} \end{cases}$

6. **20 points** Construct a state machine with input and output set $U = \{0, 1\}$, and output set $Y = \{t, f\}$ such that for any input sequence $u(0), u(1), \cdots$, the output sequence is

$$y(n) = \begin{cases} t, & \text{if } (u(n-3), u(n-2), u(n-1)) = (1, 0, 1) \\ f, & \text{otherwise} \end{cases}$$

In words: the machine outputs t if the three previous inputs are 101, otherwise it outputs f.

What is the output sequence of your machine when the input sequence is $010110101 \cdots$?

7. **20 points** A single-input, single-output difference equation system is of the form:

$\forall t = 0, 1, \cdots$

$$\begin{aligned} x(t+1) &= Ax(t) + bu(t) \\ y(t) &= c'x(t) \end{aligned}$$

where A is a $n \times n$ matrix, b and c are n-dimensional column vectors. Suppose the initial state is x_0.

(a) Write the general expression for the output sequence $y(t), t = 0, 1, \cdots$ when the input sequence is $u(0), u(1), \cdots$.

(b) Suppose that

— when the input sequence is $\forall t,\ u(t) = 1$ and the initial state is x_0, the output sequence is $\forall t,\ y(t) = 1$, and

— when the input sequence is $\forall t,\ u(t) = 1$ and the initial state is $2 \times x_0$, the output sequence is $\forall t,\ y(t) = 2$

i. Give an expression and a sketch of the output response when the initial state is x_0 and the input sequence is $\forall t,\ u(t) = 0$?

ii. Given an expression and a sketch of the output response when the initial state is 0 and the input sequence is $\forall t,\ u(t) = 1$?