EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2
Department of Electrical Engineering and Computer Sciences 25 October 2011
University of California Berkeley

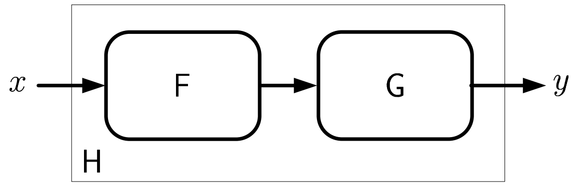
LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

MT2.1 (30 Points) A cascade (series) interconnection of two discrete-time LTI systems F and G having impulse response f and g, respectively, is shown below:



It's fairly straightforward to show that the overall system H is also LTI and that its impulse response is $h(n) = (f * g)(n)$, for all n.

Parts (a) and (b) of this problem are independent, so you may approach them in either order. This also means that particular system properties assumed in one part may *not* be carried over to the other.

(a) (15 Points) True or False? If each of the systems F and G is BIBO stable, then H must be BIBO stable.

If your answer is that the claim is true, prove the stability of H.

If your answer is that the claim is false, find a pair of BIBO stable systems F and G and show that H is *not* BIBO stable.

(b) (15 Points) True or False? It's *impossible* to find a causal system F and a non-causal system G such that $(f * g)(n)$ could serve as the impulse response of a causal LTI system $\widehat{\mathsf{H}}$.

Note that if a causal $\widehat{\mathsf{H}}$ is found, it means that a causal equivalent to H exists. It does *not* mean that H is causal. As described, H cannot be causal because at least one of its components must peek into the future of its input.

If your answer is that the claim is true, prove the nonexistence of causal equivalent to H. If your answer is that the claim is false, find a causal F and a non-causal G such that $\widehat{h} = f * g$ can be considered the impulse response of a causal system $\widehat{\mathsf{H}}$.

MT2.2 (50 Points) A causal, BIBO stable discrete-time LTI system H is characterized by the linear, constant-coefficient difference equation

$$y(n) = 2\alpha\, y(n-1) - \alpha^2 y(n-2) + x(n),$$

where $|\alpha| < 1$.

(a) (15 Points) Show that the frequency response of the system is

$$H(\omega) = \frac{1}{(1 - \alpha e^{-i\omega})^2}, \quad \text{for all } \omega.$$

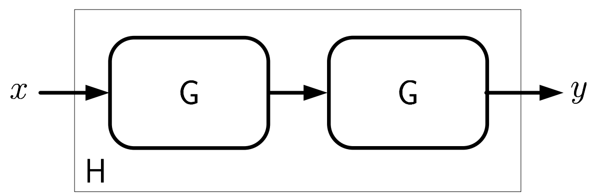
(b) (15 Points) Determine a reasonably simple expression for $h(n)$, the impulse response values of the system H.

You may or may not find the following power series expansion useful:

$$\sum_{n=0}^{+\infty} (n+1)\zeta^n = 1 + 2\zeta + 3\zeta^2 + \cdots + (n+1)\zeta^n + \cdots = \frac{1}{(1-\zeta)^2}, \quad \text{if } |\zeta| < 1.$$

(c) (10 Points) Let $\alpha = 1/2$. Determine the output of the filter H in response to the input $x(n) = 1 + (-1)^n$, for all n.

(d) (10 Points) It's straightforward to show that the system H can be implemented as the cascade (series) interconnection of two identical causal, BIBO stable LTI systems as shown below:



Determine $G(\omega)$ and $g(n)$, the frequency response and impulse response, respectively, of the system G. If $\alpha = 1/2$, provide a well-labeled approximate sketch of $|H(\omega)|$, the magnitude response of the cascade system H.

MT2.3 (25 Points) Consider a causal, finite-duration impulse response (FIR) filter H whose impulse response is characterized by

$$h(n) = \begin{cases} h(M-n) & 0 \leq n \leq M \\ 0 & \text{elsewhere.} \end{cases}$$

Assume that M is a positive, odd integer. Show that the frequency response of the filter can be written as

$$H(\omega) = A(\omega)\, e^{-i\omega M/2},$$

where

$$A(\omega) = \sum_{k=1}^{(M+1)/2} 2h\left(\frac{M+1}{2} - k\right) \cos\left[\omega\left(k - \frac{1}{2}\right)\right].$$

Also show that $A(\omega)$ is odd about $\omega = \pi$ and that it's periodic with period 4π.

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Problem	Points	Your Score
Name	10	
1	30	
2	50	
3	25	
Total	**115**	