EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 3
Department of Electrical Engineering and Computer Sciences 9 December 2008
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ____________________ FIRST Name ____________________

Lab Time ____________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT3.1 (20 Points) This entire problem is restricted to the space of continuous-time signals that are periodic with fundamental period p and fundamental frequency $\omega_0 = 2\pi/p$. Each signal in this space may be complex-valued. Otherwise, the two parts of this problem are independent, so you may tackle them in either order.

(a) True or false? $||f+g||^2 + ||f-g||^2 = 2||f||^2 + 2||g||^2$.
Explain your reasoning.

Recall that $||f||^2 \triangleq \langle f, f \rangle \triangleq \displaystyle\int_{\langle p \rangle} f(t)\, f^*(t)\, dt$ and that the magnitude-squared of other such periodic functions is similarly defined.

(b) Consider a signal x having the exponential Fourier series expansion $x(t) = \sum_{k=-\infty}^{\infty} X_k\, e^{ik\omega_0 t}$. Let

$$x_N(t) = \sum_{k=-N}^{+N} X_k\, e^{ik\omega_0 t}$$

be an approximation to x, and let $\varepsilon_N(t) = x(t) - x_N(t)$ denote an error signal. Prove that $\varepsilon_N \perp x_N$.

MT3.2 (40 Points) Consider a periodic discrete-time signal x having fundamental period p and an exponential discrete Fourier series expansion

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n},$$

where ω_0 is the fundamental frequency of the signal. The signal has the following the properties:

- $x(n+3) = x(n), \ \forall n \in \mathbb{Z}$.
- $\displaystyle\sum_{n=\langle p \rangle} x(n) = 0$.
- $\displaystyle\sum_{k=\langle p \rangle} X_k = 0$.
- $\displaystyle\sum_{k=\langle p \rangle} |X_k|^2 = \frac{1}{2}$.

Show that the signal x can be expressed as $x(n) = A\cos(Bn + C)$, and determine the parameters A, B, and C.

Is there a unique answer? If so, explain. If not, determine at least two possible signals x and provide a well-labeled stem plot for each.

MT3.3 (45 Points) Consider a discrete-time LTI filter H having impulse response h and frequency response H.

Recall that $H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-i\omega n}$.

A related discrete-time filter G has impulse response

$$g(n) = h(n) - h(n-1).$$

(a) Determine a simple expression for the frequency response values $G(\omega)$ in terms of $H(\omega)$.

(b) Suppose the frequency response of the filter H is given by

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \frac{1}{1 - 0.99\, e^{-i\omega}}.$$

(i) Determine a simple expression for its impulse response $h(n)$.

(b) Continuation of part (b) from the previous page:

(ii) Determine a simple expression for the frequency response $G(\omega)$, and provide a well-labeled plot of the magnitude response $|G(\omega)|$. Specify whether the filter G is low-pass, band-pass, high-pass, notch, or none of these types. Explain your work.

(iii) Determine the linear, constant-coefficient difference equation that governs the input x and output y of the filter G.

(iv) Determine a reasonably simple and accurate expression, as well as a well-labeled stem plot, for the response of the filter G to the input signal $x(n) = 1 + (-1)^n$? Explain.

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Problem	Points	Your Score
Name	10	
1	20	
2	40	
3	45	
Total	**115**	