EECS 20N: Structure and Interpretation of Signals and Systems Final Exam
Department of Electrical Engineering and Computer Sciences 13 December 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Please print your name and lab time in legible, block lettering above, and on the back of the last page.
- This exam should take you about two hours to complete. However, you will be given up to about three hours to work on it. We recommend that you budget your time as a function of the point allocation and difficulty level (for you) of each problem or modular portion thereof.
- **This exam printout consists of pages numbered 1 through 14. Also included is a double-sided appendix sheet containing transform properties.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the fourteen numbered pages and the appendix. If you find a defect in your copy, notify the staff immediately.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, to receive full credit, you must explain your responses succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.
- It has been a pleasure having you in EECS 20N. Happy holidays!

- Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \ ,$$

  where $p = \frac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable contiguous discrete interval of length p (for example, $\sum_{k=\langle p \rangle}$ can denote $\sum_{k=0}^{p-1}$).

- Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period p:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ik\omega_0 t} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt \ ,$$

  where $p = \frac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable continuous interval of length p (for example, $\int_{\langle p \rangle}$ can denote $\int_0^p$).

- Discrete-time Fourier transform (DTFT) synthesis and analysis equations for a discrete-time signal:

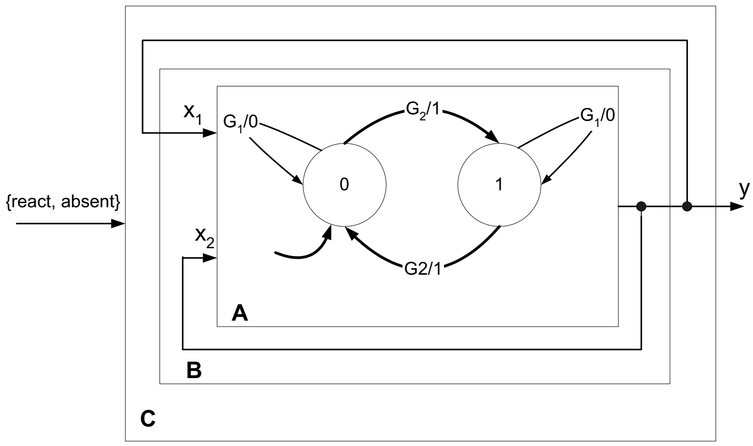
$$x(n) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\omega) e^{i\omega n} d\omega \qquad \longleftrightarrow \qquad X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n} \ ,$$

  where $\langle 2\pi \rangle$ denotes a suitable continuous interval of length 2π (for example, $\int_{\langle 2\pi \rangle}$ can denote $\int_0^{2\pi}$ or $\int_{-\pi}^{\pi}$).

- Continuous-time Fourier transform (CTFT) synthesis and analysis equations for a continuous-time signal:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \qquad \longleftrightarrow \qquad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \ .$$

F05.1 (20 Points) Consider the finite-state machine composition shown below:



Let the set $\mathsf{D} = \{0, 1, \mathsf{absent}\}$ denote an alphabet. For every pair $(x_1(n), x_2(n)) \in \mathsf{D}^2$, $x_1(n)$ and $x_2(n)$ denote the top and bottom input symbols in the figure, respectively. The n^{th} output symbol $y(n) \in \mathsf{D}$.

For *each* of the following guard sets G_1, G_2, and for each of the machines B and C, determine whether the machine is well-formed (WF) or not well-formed (NWF) by circling one choice (WF or NWF) in each entry of the table below? No explanation will be considered. No partial credit will be given.

$$(I) \begin{cases} G_1 = \{(1,0)\} \\ G_2 = \{(0,1)\} \end{cases} \qquad (II) \begin{cases} G_1 = \{(0,0),(1,0)\} \\ G_2 = \{(0,1),(1,1)\} \end{cases}$$

$$(III) \begin{cases} G_1 = \{(1,1)\} \\ G_2 = \{(0,0)\} \end{cases} \qquad (IV) \begin{cases} G_1 = \{(0,0),(1,0)\} \\ G_2 = \{\} \end{cases}$$

Guard Set	Machine B	Machine C
(I)	WF NWF	WF NWF
(II)	WF NWF	WF NWF
(III)	WF NWF	WF NWF
(IV)	WF NWF	WF NWF

F05.2 (40 Points) [N-Fold Upsampler] Consider a discrete-time system whose input and output signals are denoted by $x : \mathbb{Z} \to \mathbb{R}$ and $y : \mathbb{Z} \to \mathbb{R}$, respectively. The output y is obtained by upsampling the input x by a factor of N, where $N \in \{2, 3, \ldots\}$. That is,

$$\forall n \in \mathbb{Z}, \quad y(n) = \begin{cases} x\left(\dfrac{n}{N}\right) & \text{if } n \bmod N = 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Select the strongest assertion from the choices below. Explain your choice.

(I) The system must be time invariant.

(II) The system could be time invariant.

(III) The system cannot be time invariant.

(b) Select the strongest assertion from the choices below. Explain your choice.

(I) The system must be causal.

(II) The system could be causal.

(III) The system cannot be causal.

(c) Select the strongest assertion from the choices below. Explain your choice.

(I) The system must be memoryless.

(II) The system could be memoryless.

(III) The system cannot be memoryless.

(d) Suppose the input signal x is periodic with fundamental frequency $\omega_0 = 2\pi/p$, where p denotes the period, and has the discrete Fourier series (DFS) expansion

$$x(n) = \sum_{k=\langle p\rangle} X_k \, e^{ik\omega_0 n} \, .$$

(i) Determine the period $\widehat{p}$ and the corresponding fundamental frequency $\widehat{\omega}_0$ of the periodic output signal y. Your answers must be in terms of p and ω_0.

(ii) Determine the DFS coefficients $Y_k, k \in \{0, 1, \ldots, \widehat{p} - 1\}$, in terms of the DFS coefficients X_k of the input signal.

Note: You can approach this problem in more than one way. Depending on which method you use, you may or may not need the following nuggets (δ denotes the Dirac delta function):

$$e^{i\omega_0 n} \overset{\text{DTFT}}{\longleftrightarrow} 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega - \omega_0 + 2\pi r)$$

$$\delta(\alpha(\mu - \mu_0)) = \frac{1}{|\alpha|}\delta(\mu - \mu_0) \, .$$

(e) Suppose $N = 2$ and $x(n) = \cos\left(\frac{\pi}{2}n\right), \forall n \in \mathbb{Z}$. Let $w : \mathbb{Z} \to \mathbb{R}$ denote the impulse response of a discrete-time low-pass LTI filter, whose frequency response is defined as follows:

$$\forall \omega \in \mathbb{R}, \quad W(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{2} \\ 0 & \text{elsewhere.} \end{cases}$$

If the upsampled signal y is processed by the LTI filter to produce the signal $v : \mathbb{Z} \to \mathbb{R}$, determine which of the following choices best characterizes v ($K \neq 0$ denotes a real constant whose value is not of concern to us here).

(I)

$$v(n) = K \cos\left(\frac{3\pi}{4}n\right), \forall n \in \mathbb{Z}.$$

(II)

$$v(n) = K \cos\left(\frac{\pi}{6}n\right), \forall n \in \mathbb{Z}.$$

(III)

$$v(n) = K \cos\left(\frac{\pi}{4}n\right), \forall n \in \mathbb{Z}.$$

(IV)

$$v(n) = K \cos\left(\frac{2\pi}{3}n\right), \forall n \in \mathbb{Z}.$$

Explain your reasoning succinctly, but clearly and convincingly.

F05.3 (30 Points) Consider a finite-length continuous-time signal $x : \mathbb{R} \to \mathbb{R}$ whose region of support is confined to the interval $(-T, +T)$, where $T > 0$. That is, $x(t) = 0$, if $|t| > T$. Let $X : \mathbb{R} \to \mathbb{C}$ denote the continuous-time Fourier transform (CTFT) of x, i.e.,

$$\forall \omega \in \mathbb{R}, \quad X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt\,.$$

Suppose the function $X(\omega)$ is modulated by the frequency-domain impulse train S, where

$$S(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)\,,$$

where $\omega_s = 2\pi/T_s$. Let the resulting function be denoted by Y, where $Y(\omega) = X(\omega)\, S(\omega)$. In effect, we are sampling the CTFT of x here.

(a) Determine y, the inverse Fourier transform of Y. Sketch a sample signal x and show how y is related to x.

(b) What condition(s) must x satisfy so that it is recoverable from this process of frequency-domain sampling? That is, under what condition(s) (imposed on x) can we recover x from y. Explain how y should be processed to yield x.

F05.4 (40 Points) Consider a discrete-time signal x. Each part below discloses partial information about x. Ultimately, your task is to determine x completely.

In the space provided for each part, state and explain every inference that you can draw about x, synthesizing information disclosed, or your own inferences drawn, up to, and including, that part. Justify all your work succinctly, but clearly and convincingly.

(a) The signal x coincides with, and is equal to, exactly one period of a real-valued periodic signal $\widetilde{x} : \mathbb{Z} \to \mathbb{R}$. It is known that the fundamental frequency of $\widetilde{x}$ is

$$\omega_0 = \frac{2\pi}{5} \, .$$

(b) The following is known about X, the discrete-time Fourier transform (DTFT) of x:

(i) $X(\omega) \in \mathbb{R}, \forall \omega \in \mathbb{R}$.

(ii) $X(\omega)\big|_{\omega=0} = \dfrac{3}{2} \, .$

(iii) $X(\omega)\big|_{\omega=\pi} = \frac{5}{2}\,.$

(iv) $\int_0^{2\pi} X(\omega)\, d\omega = 2\pi.$

- Determine, and provide a well-labeled plot of, the signal x.

F05.5 (40 Points) A function $f : \mathbb{R} \to \mathbb{C}$, which we call a *"mother wavelet,"* has the following properties:

1. f has zero average, i.e.,
$$\int_{-\infty}^{\infty} f(t)\, dt = 0\,.$$
2. f has finite energy, i.e.,
$$\mathcal{E}_f \triangleq ||f||^2 \triangleq \langle f, f \rangle \triangleq \int_{-\infty}^{\infty} f(t)\, f^*(t)\, dt = \int_{-\infty}^{\infty} |f(t)|^2\, dt < \infty.$$
In fact, throughout this problem, assume, without loss of generality, that f is normalized to have unit energy, i.e., $\mathcal{E}_f = 1$.

Consider a family of *"offspring wavelets"* (also called *"atoms"*) obtained by time-scaling and time-shifting f:

$$f_{\alpha,\tau}(t) = \frac{1}{\sqrt{\alpha}}\, f\left(\frac{t-\tau}{\alpha}\right)\ ,$$

where $\alpha \in \mathbb{R}^+$ ($\mathbb{R}^+$ denotes the set of positive real numbers) and $\tau \in \mathbb{R}$.

(a) Suppose the mother wavelet f denotes the impulse response of a linear, time-invariant (LTI) filter. Select the strongest assertion from the choices below. Explain your reasoning succinctly, but clearly and convincingly.

(I) f could represent a low-pass filter.
(II) f must represent a low-pass filter.
(III) f could represent a band-pass filter.
(IV) f must represent a band-pass filter.
(V) f could represent a high-pass filter.
(VI) f must represent a high-pass filter.

(b) Determine $F_{\alpha,\tau} : \mathbb{R} \to \mathbb{C}$, the continuous-time Fourier transform (CTFT) of $f_{\alpha,\tau}$. That is, determine an expression for

$$F_{\alpha,\tau}(\omega) \triangleq \int_{-\infty}^{\infty} f_{\alpha,\tau}(t)\, e^{-i\omega t}\, dt\,.$$

(c) Consider the *Haar Family* of wavelet functions, defined by:

$$f_{2^m,n}(t) = \frac{1}{\sqrt{2^m}}\, f\left(\frac{t - 2^m n}{2^m}\right), \qquad m, n \in \mathbb{Z}\,.$$

The time-scale factor is denoted by m and the time-shift factor by n. *In what follows, assume $n = 0$.*
The mother wavelet f corresponds to $m = n = 0$, i.e.,

$$f(t) \triangleq f_{2^0,0}(t) = \begin{cases} +1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

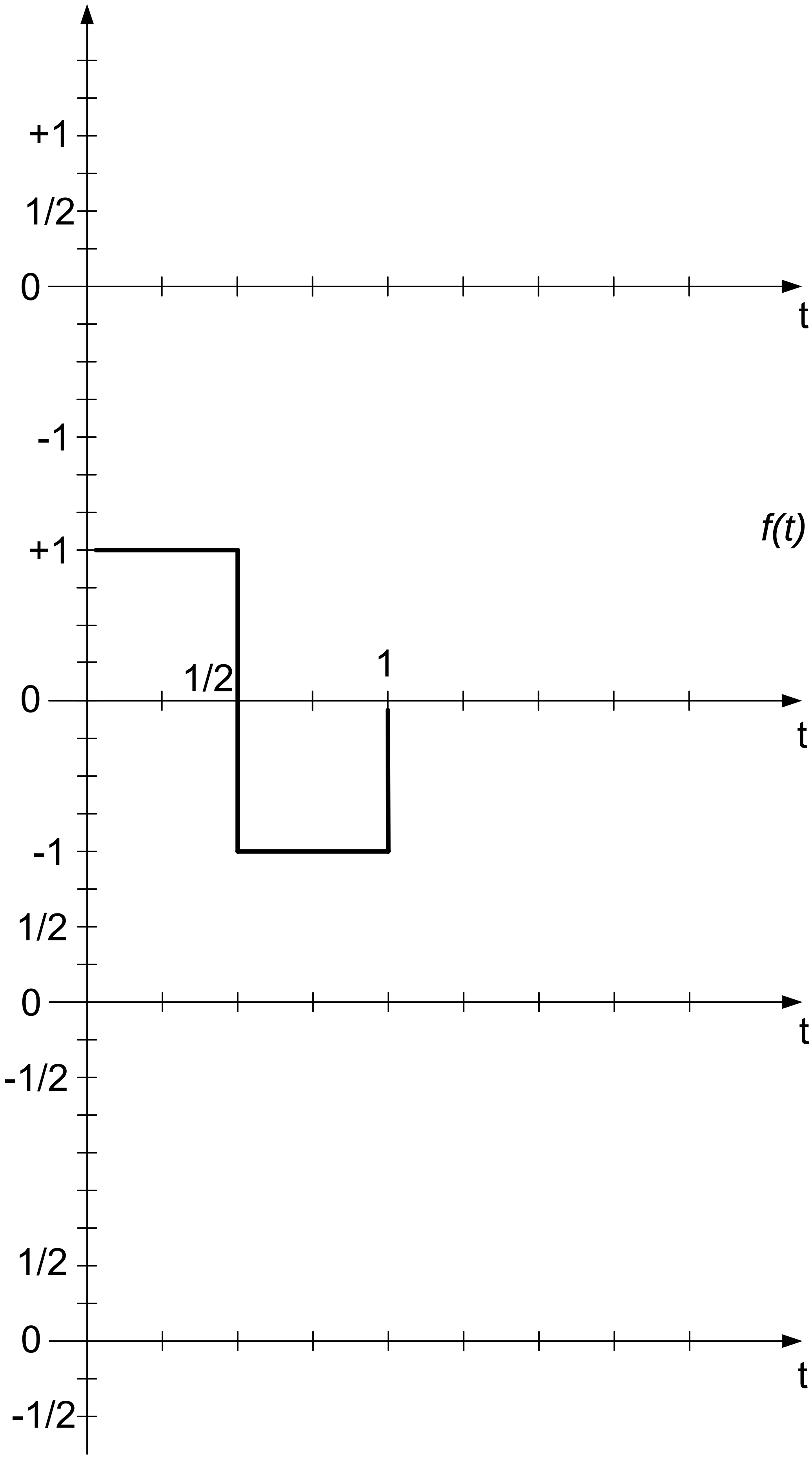
(i) Without complicated mathematical manipulation, determine the energy and the average of each Haar atom $f_{2^m,n}$.

(ii) Plotted on the next page is a sketch of the Haar mother wavelet f. In the other spaces, provide well-labeled plots of the unshifted Haar wavelet atoms characterized by $m = -1, +1, 2$, respectively.

(iii) Explain why the Haar wavelets $\{f_{2^m,0}\}_{m\in\mathbb{Z}}$ are mutually orthogonal, i.e.,

$$\langle f_{2^m,0}, f_{2^k,0}\rangle \triangleq \int_{-\infty}^{\infty} f_{2^m,0}(t)\, f^*_{2^k,0}(t)\, dt = \delta(k-m).$$

We are not looking for a rigorous mathematical proof here. You should be able to infer mutual orthogonality by observing features of the plots that you drew above and exploiting one of the salient properties of f given in the problem statement.


+1
1/2
0
t
-1
f(t)
+1
1/2
1
0
t
-1
1/2
0
t
-1/2
1/2
0
t
-1/2

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

Problem	Points	Your Score
Name	10	
1	20	
2	40	
3	30	
4	40	
5	40	
Total	**180**	