EECS 20. Midterm No. 2

November 19, 2004. Use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: ______________________________

Lab day and time: ______________________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Total:

1. **20 points** A system is described by the difference equation

$$y(n) = x(n) + x(n-1) + 0.5y(n-1).$$

(a) **7 points** Obtain the $[A, b, c^T, d]$ representation of this system by:

i. choosing the state,

ii. calculating A, b, c^T, d for your choice of state.

(b) **8 points** If $x(-1) = 0, y(-1) = 1$, calculate the zero-input (i.e. $x(n) = 0, n \geq 0$) state response.

(c) **5 points**

Calculate the frequency response H of this system. What are the magnitude and phase of H at $\omega = 0, \pi$?

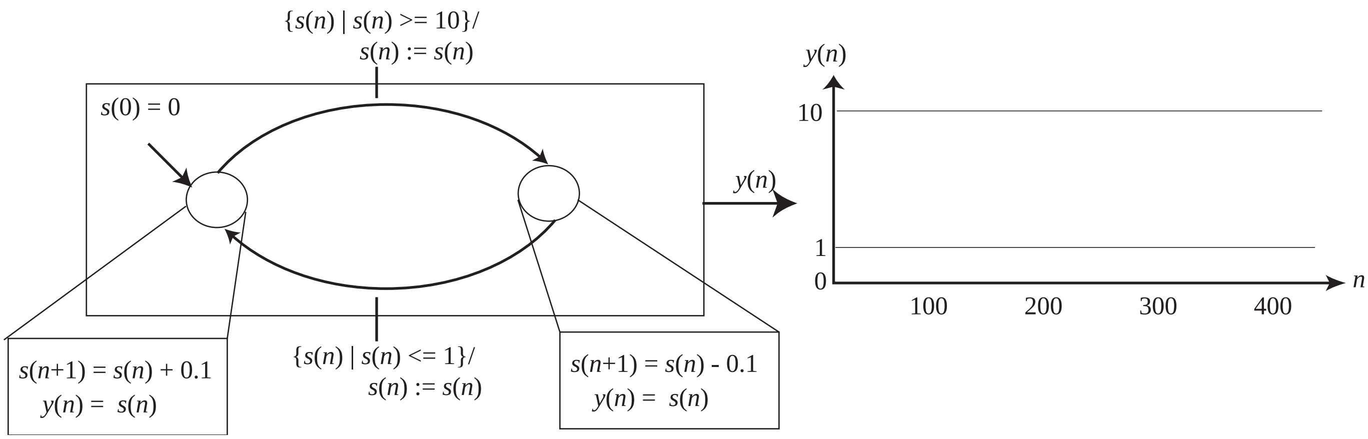

Figure 1: Hybrid system for problem 2

2. **10 points**

   Consider the hybrid system of fig 1.

   (a) **5 points** For $0 \leq n \leq 400$, write down an expression for $s(n)$ and then sketch the output $y(n)$. Carefully mark the times when y reaches its maximum and minimum values.

   (b) **5 points** Does y become periodic after some finite time? If yes, what is the period?

3. **20 points, 5 points each part** For each of the following discrete-time systems S, state whether it is time-invariant (TI), linear (L), causal (C) and memoryless (ML). For each system prove your answer for the property indicated in **bold**.

   (a) $\forall x, n, \quad S(x)(n) = x(n-1) + x(n)$. **Causal?**.

   (b) $\forall x, n, \quad S(x)(n) = x(2n)$. **Time-invariant?**.

   (c) $\forall x, n, \quad S(x)(n) = [x(n-1)]^2$. **Memoryless?**.

   (d) $\forall x, n, \quad S(x)(n) = 2nx(n-1)$. **Linear?**.

4. **15 points, 5 points each part** A continuous-time LTI system has frequency response

$$\forall \omega, H(\omega) = [1 + i\omega]^{-1}.$$

(a) Plot its magnitude and phase response. Mark the values for $\omega = 0, 1$

(b) The frequency response of another LTI system is

$$\forall \omega, G(\omega) = [H(\omega)]^2.$$

Plot its magnitude and phase response. Mark the values for $\omega = 0, 1$.

(c) The signal $\forall t, x(t) = \cos(t) + \cos(10t) + \cos(100t)$ is input to G. Calculate the output, making the approximation $1 + i\omega \approx i\omega$ for $|\omega| > 9$.

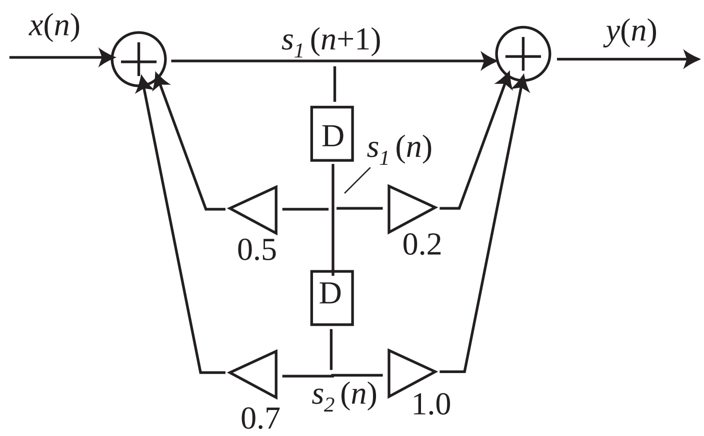

Figure 2: System for problem 5

5. **15 points** You are given three kinds of building blocks for discrete-time systems: one-unit delay; gains; and adders.

   (a) **5 points** Use these building blocks to implement the system:

   $$y(n) = 0.5y(n-2) + x(n) + x(n-1).$$

   (b) **10 points** For the system of figure 2 obtain its $[A, b, c^T, d]$ representation taking the two-dimenstional state at time n to be $s(n) = [s_1(n), s_2(n)]^T$, the output of the two delays.

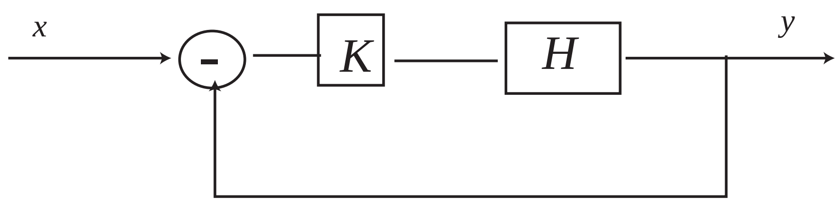

Figure 3: Feedback system for problem 6

6. **20 points** In the negative feedback system of figure 3 assume that $H(\omega) = [1 + i\omega]^{-1}$. Let G_K be the closed-loop frequency response. For $K = 1, 10, 100$

   (a) **10 points** Determine $G_K(\omega), |G_K(\omega)|, \angle G_K(\omega)$. Draw two plots: one for all the magnitude responses $|G_K(\omega)|$, and another for all the phase responses $\angle G_K(\omega)$.

   (b) **5 points** Determine ω_K at which $\angle G_K(\omega_K) = -\pi/4$.

   (c) **5 points** Determine the response y_K of G_K to the input signal $\forall t, x_K(t) = \cos(\omega_K t)$