

**EECS 20. Final Exam**  
**December 19, 2002.**

Please use these sheets for your answer. **Write clearly and show your work.** Please check that you have 12 numbered pages.

Print your name and lab time below

Name: \_\_\_\_\_

Lab time: \_\_\_\_\_

Problem 1 (12):

Problem 2 (16):

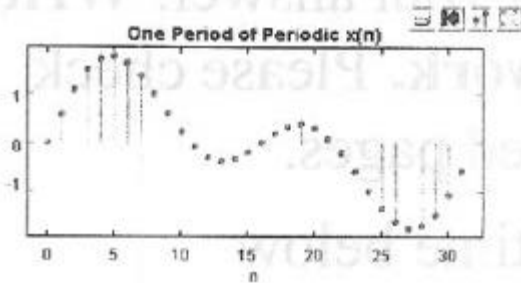
Problem 3 (28):

Problem 4 (24):

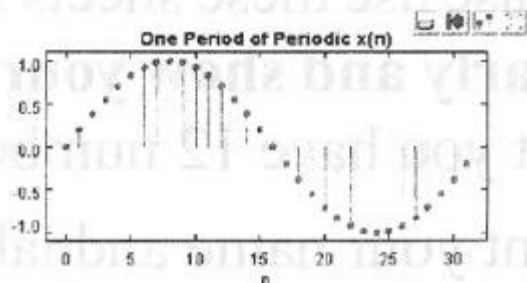
Problem 5 (20):

Total:

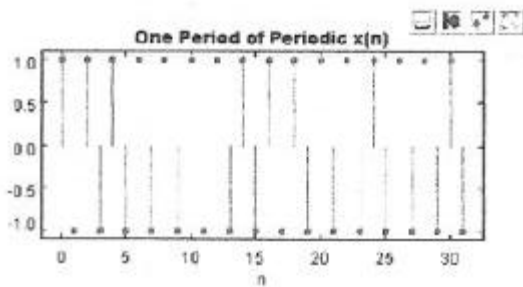
1. **12 points.** Suppose that  $x$  is a discrete-time signal with period  $p = 32$ . Below are plotted six possible such signals  $x$ . For each of these, match one of the six plots on the next page, or indicate that none matches.



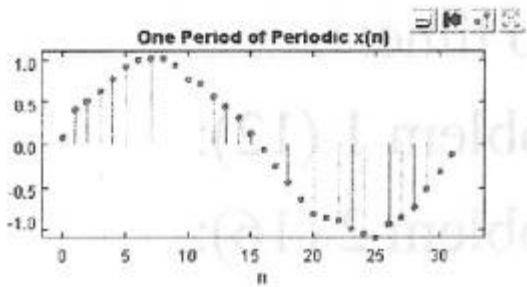
matching FS: \_\_\_\_\_



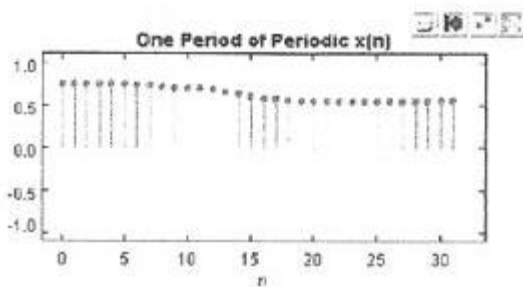
matching FS: \_\_\_\_\_



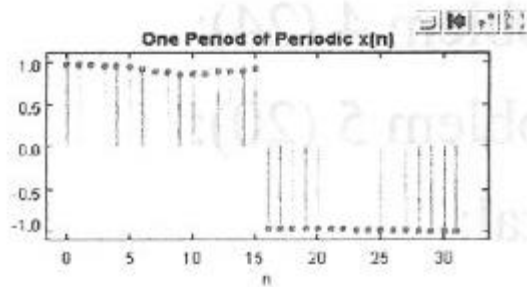
matching FS: \_\_\_\_\_



matching FS: \_\_\_\_\_

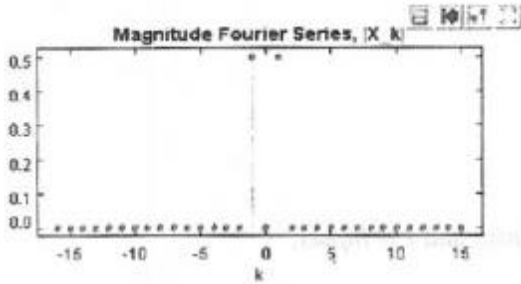


matching FS: \_\_\_\_\_

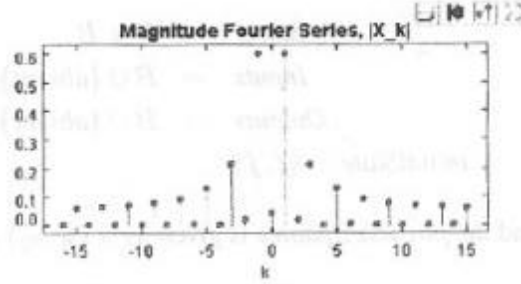


matching FS: \_\_\_\_\_

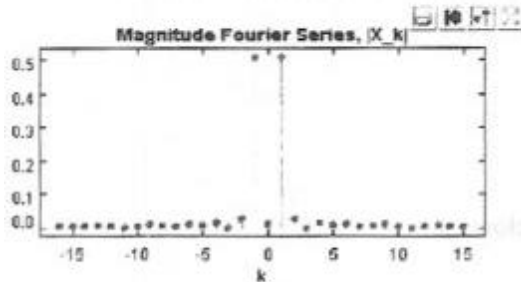
Below are six plots of the magnitude  $|X_k|$  of the Fourier series coefficients  $X_k$  of a periodic signal  $x$  with period  $p = 32$ , for six different such periodic signals.



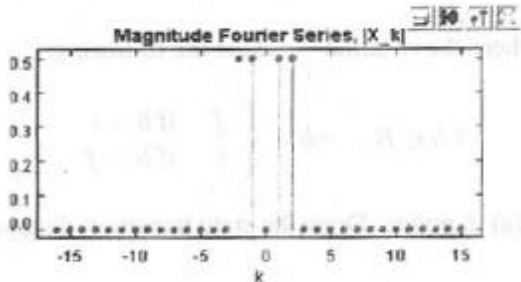
(a)



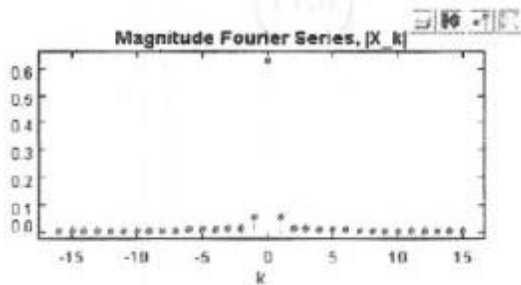
(b)



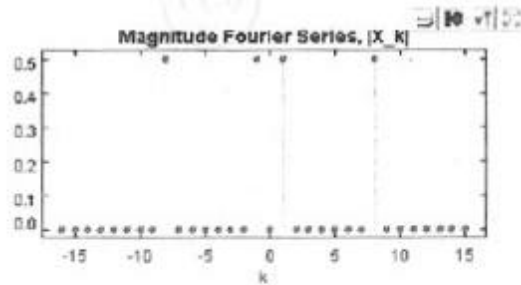
(c)



(d)



(e)



(f)

2. **16 points.** Let  $B = \{t, f\}$  be the set of binary truth values. Consider a nondeterministic state machine with

$$\begin{aligned} \text{States} &= B \times B \\ \text{Inputs} &= B \cup \{\text{absent}\} \\ \text{Outputs} &= B \cup \{\text{absent}\} \\ \text{initialState} &= (f, f) \end{aligned}$$

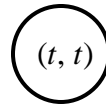
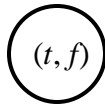
and the *possibleUpdates* is given by  $\forall (s_1, s_2) \in \text{States}$  and  $i \in \text{Inputs}$ ,

$$\text{possibleUpdates}((s_1, s_2), i) = \begin{cases} \{(s_1, s_2), \text{absent}\} & \text{if } i = \text{absent} \\ \{(s_1, s_2), s_1\}, (s_1, \neg s_2), s_1\} & \text{if } i = f \\ \{(\neg s_1, s_2), s_1\}, (\neg s_1, \neg s_2), s_1\} & \text{if } i = t \end{cases}$$

where the notation  $\neg$  means the following:

$$\forall b \in B, \neg b = \begin{cases} f & \text{if } b = t \\ t & \text{if } b = f \end{cases}$$

(a) 6 points. Draw the state transition diagram below.



(b) 4 points. For the input sequence  $(f, f, t, t, f, f, \dots)$ , give one possible output sequence. You need show only the first six output symbols.

(c) 6 points. Give a simpler deterministic state machine that is bisimilar to this one, and give the bisimilarity relation.

3. **28 points.** Determine whether each of the following statements is true or false. There will be no partial credit, so consider your answer carefully.

(a)  $\forall t \in \text{Reals}, \quad \cos(\mathbf{p}t) = (-1)^t$ .

(b) If the input to a continuous-time LTI system is  $x$  such that

$$\forall t \in \text{Reals}, \quad x(t) = \cos(t) + \sin(t),$$

then  $\exists A \in \text{Reals}$  and  $\exists \mathbf{f} \in \text{Reals}$  such that the output  $y$  of the LTI system is

$$y(t) = A \cos(t + \mathbf{f}).$$

(c)  $\{[1,2] \rightarrow [1,2]\} \subset \{[1,2] \rightarrow \text{Naturals}\}$ .

(d) Suppose  $[A, b, c, d]$  and  $[A', b', c', d']$  describe two different LTI systems. If the frequency response of the two is the same, then it must be that  $A = A'$ .

(e) An identity function is  $f : X \rightarrow X$  where  $\forall x \in X, f(x) = x$ , suppose that  $f : X \rightarrow X$  and  $g : Y \rightarrow Y$  are both identity functions, and further that  $X \subset Y$ . Then  $\text{graph}(f) \subset \text{graph}(g)$ .

(f) For any sets  $X$  and  $Y$ ,  $X \in P(X \cup Y)$ , where  $P$  denotes the powerset.

(g) If  $y$  is a continuous-time signal given by

$$\forall t \in \text{Reals}, \quad y(t) = \sum_{k=-\infty}^{\infty} \mathbf{d}(t-k),$$

where  $\mathbf{d}$  is the Dirac delta function, then  $\forall t \in \text{Reals}$  where  $t \notin \text{Integers}$ ,  $y(t) = 0$ .

4. **24 Points.** Consider a discrete-time function  $x$  where

$$\forall n \in \text{Integers}, \quad x(n) = 1 + \cos(pn/4) + \sin(pn/2)$$

(a) Find the period  $p$  and fundamental frequency  $\omega_0$ . Give the units.

(b) Find  $K$  and the Fourier series coefficients  $A_0, \dots, A_k$  and  $\mathbf{f}_0, \dots, \mathbf{f}_k$  in

$$x(n) = A_0 + \sum_{k=1}^K A_k \cos(k\omega_0 n + \mathbf{f}_k).$$



(c) Find the Fourier series coefficients  $X_k$  in

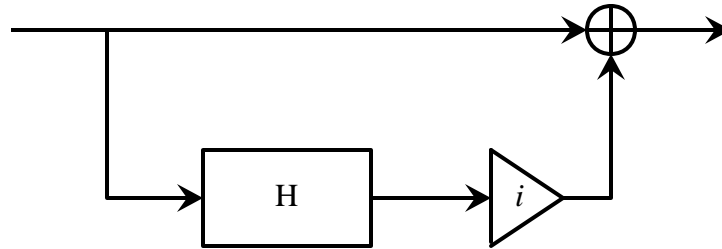
$$x(n) = \sum_{k=0}^{p-1} X_k e^{ikw_0 n}$$

(d) Let  $x$  be the input to an LTI system with frequency response  $H$  given by

$$\forall \mathbf{w} \in \text{Reals}, H(\mathbf{w}) = \begin{cases} -i & \text{if } 0 < \mathbf{w} < p \\ 0 & \text{if } \mathbf{w} = 0 \text{ or } \mathbf{w} = p \\ i & \text{if } -p < \mathbf{w} < 0 \end{cases}$$

For other values of  $\mathbf{w}$ ,  $H(\mathbf{w})$  is determined by the periodicity of  $H$ . (Such a system is called a Hilbert filter.) Find the output.

(e) Find the frequency response  $H'$  of a new system constructed as follows:



where  $H$  is the Hilbert filter from part (d), and the triangle with an  $i$  scales its input by the imaginary number  $i$ . Note that even if the input is real-valued, the output is likely to be complex-valued. Such a system is called a Hilbert transformer, approximations to it are widely used in digital communication systems.

(f) Let the input to the system in part (e) be  $x$ . What is the output?

5. **20 points.** Let the continuous-time signal  $c$  given by

$$\forall t \in \text{Reals}, \quad c(t) = 2 \cos(\mathbf{w}_c t)$$

be a carrier wave for a radio signal. Let  $x$  given by

$$\forall t \in \text{Reals}, \quad x(t) = 2 \cos(\mathbf{w}_x t)$$

be the signal to be carried by that radio signal (that is, it is a highly simplified stand-in for, say, a voice signal). To be concrete, let  $\mathbf{w}_c = 2\mathbf{p} \cdot 8000$  radians/second, and  $\mathbf{w}_x = 2\mathbf{p} \cdot 400$  radians/second.

(a) Find and sketch the CTFT of  $y$  where

$$\forall t \in \text{Reals}, \quad y(t) = c(t)x(t).$$

Label your sketch carefully. **Hint:** The CTFT of  $e^{i\mathbf{w}_0 t}$  is  $2\mathbf{p}d(\mathbf{w} - \mathbf{w}_0)$ .

(b) Let  $y$  from part (a) be the input to an LTI system with frequency response  $H$  where

$$\forall t \in \text{Reals}, \quad H(\mathbf{w}) = \begin{cases} 0 & \text{if } \mathbf{w} \leq 0 \\ 1 & \text{if } \mathbf{w} > 0 \end{cases}.$$

Find the output  $u$ .

(c) For the same  $u$  from part (b), let

$$u' = \text{Sampler}_T(u).$$

Where  $T = 1/8000$  seconds. Find a single expression for  $u'$ .

(d) Give the signal  $z = \text{IdealInterpolator}_T(u')$ , where again  $T = 1/8000$  seconds, and  $u'$  is from part (c).