EECS 20. Midterm No. 2
November 9, 2001.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab time below

Name: ______________________________

Lab time: ____________________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. **20 points.** Consider a continuous-time signal $x \colon \textit{Reals} \to \textit{Reals}$ defined by

$$\forall\, t \in \textit{Reals}, \quad x(t) = \cos(\omega_1 t) + \cos(\omega_2 t),$$

where $\omega_1 = 2\pi$ and $\omega_2 = 3\pi$ radians/second.

(a) Find the smallest period $p \in \textit{Reals}_+$, where $p > 0$.

(b) Give the fundamental frequency corresponding to the period in (a). Give the units.

(c) Give the coefficients $A_0, A_1, A_2, \cdots$ and $\phi_1, \phi_2, \cdots$ of the Fourier series expansion for x.

2. **30 points.** Suppose that the continuous-time signal $x\colon$ *Reals* $\to$ *Reals* is periodic with period p. Let the fundamental frequency be $\omega_0 = 2\pi/p$. Suppose that the Fourier series coefficients for this signal are known constants $A_0, A_1, A_2, \cdots$ and $\phi_1, \phi_2, \cdots$. Give the Fourier series coefficients $A_0', A_1', A_2', \cdots$ and $\phi_1', \phi_2', \cdots$ for each of the following signals:

   (a) ax, where $a \in$ *Reals* is a constant

   (b) $D_\tau(x)$, where $\tau \in$ *Reals* is a constant

(c) $S(x)$, where S is an LTI system with frequency response H given by

$$\forall\, \omega \in \textit{Reals}, \quad H(\omega) = \begin{cases} 1; & \text{if } \omega = 0 \\ 0; & \text{otherwise} \end{cases}$$

(Note that this is a highly unrealistic frequency response.)

Extra Credit:

(d) Let $y\colon \textit{Reals} \to \textit{Reals}$ be another periodic signal with period p. Suppose y has Fourier series coefficients $A_0'', A_1'', A_2'', \cdots$ and $\phi_1'', \phi_2'', \cdots$. Give the Fourier series coefficients of $x + y$.

3. **30 points.** Consider discrete-time systems with input $x\colon$ *Integers* $\to$ *Reals* and output $y\colon$ *Integers* $\to$ *Reals*. Each of the following defines such a system. For each, indicate whether it is linear (L), time-invariant (TI), both (LTI), or neither (N). Note that no partial credit will be given for these questions.

   (a) $\forall\, n \in$ *Integers*, $\quad y(n) = x(n) + 0.9y(n-1)$

   (b) $\forall\, n \in$ *Integers*, $\quad y(n) = \cos(2\pi n)x(n)$

   (c) $\forall\, n \in$ *Integers*, $\quad y(n) = \cos(2\pi n/9)x(n)$

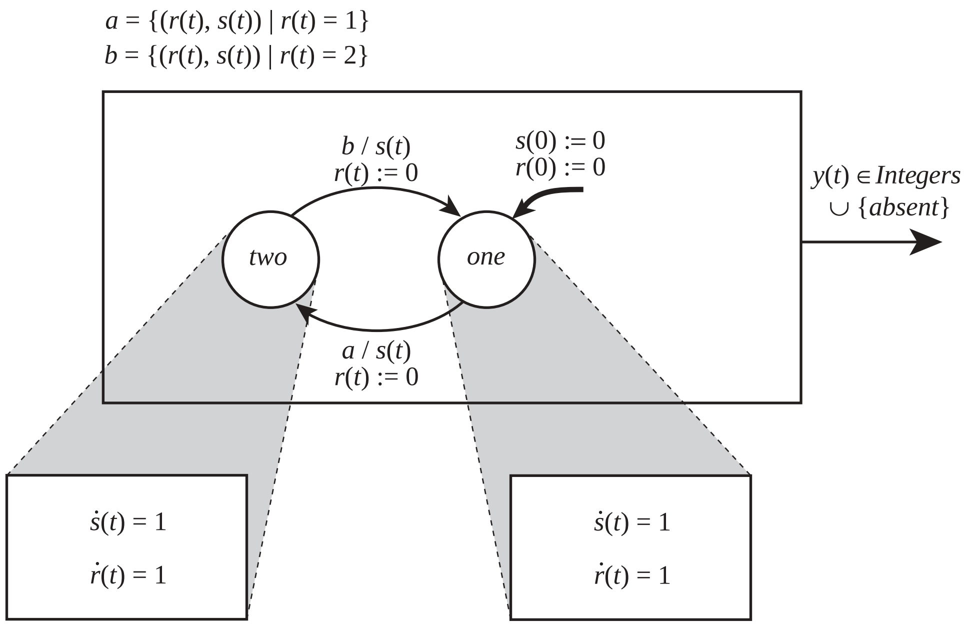
   (d) $\forall\, n \in$ *Integers*, $\quad y(n) = \cos(2\pi n/9)(x(n) + x(n-1))$

   (e) $\forall\, n \in$ *Integers*, $\quad y(n) = x(n) + 0.1(x(n))^2$

   (f) $\forall\, n \in$ *Integers*, $\quad y(n) = x(n) + 0.1(x(n-1))^2$

4. **20 points.** The objective of this problem is to understand a timed automaton, and then to modify it as specified.

   (a) For the timed automaton shown below, describe the output y. You will lose points for imprecise or sloppy notation.



(b) Assume there is a new input $u\colon Reals \to Inputs$ with alphabet

$$Inputs = \{reset, absent\},$$

and that when the input has value *reset*, the hybrid system starts over, behaving as if it were starting at time 0 again. Complete the diagram below so that it behaves like the system in (a) except that it responds to the *reset* input accordingly. Again, you will lose point for imprecise or sloppy notation. Make sure you actually complete the diagram, showing everything that needs to be shown.