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Section 1: Warm-up questions (48 points)

3. True or False (2 pts for each question) For each question below, circle **T** on the left of each statement if you think the statement is true; else circle **F** (for false).

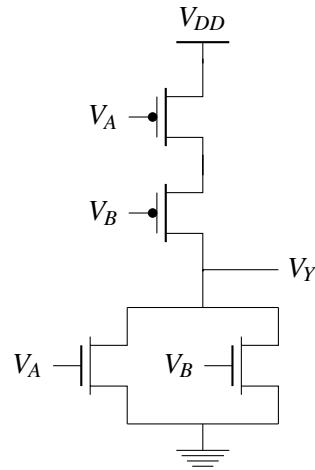
- (a) [**T / F**] An ideal capacitor dissipates energy from the circuit in the form of heat.
- (b) [**T / F**] An ideal “golden rules” op-amp behaves as though it has infinite gain.
- (c) [**T / F**] A series RLC circuit connected with a DC input voltage/current in a single loop cannot exhibit voltage or current oscillations in time.
- (d) [**T / F**] Given an impedance Z connected across a voltage source $v(t)$, it is possible for $i(t)$ to be in-phase (no phase shift) with a sinusoidal $v(t)$.
- (e) [**T / F**] Since the current across an open circuit must be zero, the voltage across the open circuit must also be zero by Ohm’s law.
- (f) [**T / F**] The voltage across a constant current source must be zero.
- (g) [**T / F**] An electrical impedance across two terminals $Z = j\omega k$ (where ω is a positive angular frequency in rad/s and k is a positive real number) can be implemented using only capacitors.

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4. Digital Circuits (9 pts)

Consider the circuit below:

- (a) (3 pts) The circuit below is a legal CMOS gate. A , B and Y are the Boolean values of the voltages, V_A , V_B and V_Y , respectively. Write down Y as a Boolean formula involving A and B .



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Operator	Meaning
\neg	NOT
\vee	OR
\wedge	AND
\oplus	XOR

Table 1: Reminder: Logical Operators

Implement each of the following Boolean functions with a single CMOS gate (i.e. implemented using a pull-up network consisting of PMOS transistors connected to a pull-down network consisting of NMOS transistors) by drawing it, or state why the function cannot be implemented as a single CMOS gate in 1-3 sentences. You only have available V_A and V_B as inputs.

(b) (3 pts) $\neg(A \wedge B)$.

(c) (3 pts) $A \wedge B$.

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5. Can you control me? (8pts)

We have a discrete time system that evolves according to $\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$. For each part, **answer whether there exists a sequence of control vectors $\vec{u}(t)$ that will bring the state to the origin $\vec{0}$ in a finite number of steps no matter where it starts.**

(a) (4 pts) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

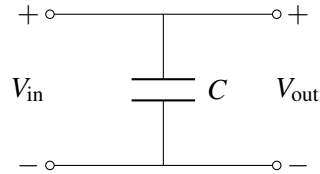
(b) (4 pts) $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

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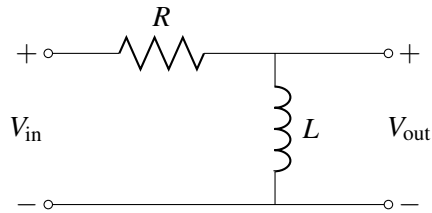
6. Transfer Functions (9 pts)

Consider the circuit diagrams below. We define $H(\omega) = V_{\text{out}}/V_{\text{in}}$ as the voltage transfer function for each circuit. Here, assume that the input is connected to an ideal voltage source that applies a sinusoidal voltage. For each circuit, **provide an expression for $H(\omega)$** where ω is the frequency of the applied sinusoidal voltage in radians per second. Here the transfer functions should be expressed as functions of j , ω , constants and the physical constants (R , C , L) of the systems.

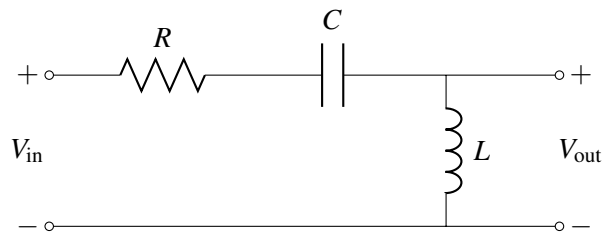
(a) (3 pts) $H(\omega) = ?$



(b) (3 pts) $H(\omega) = ?$

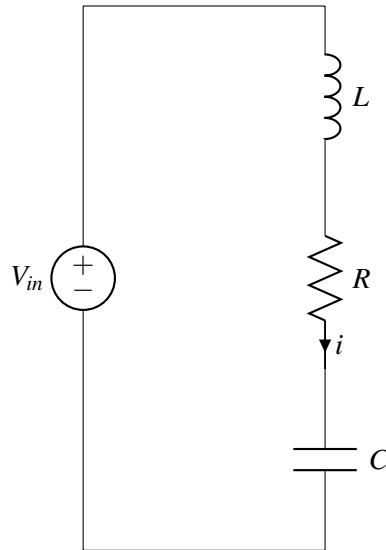


(c) (3 pts) $H(\omega) = ?$



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7. RLC Transient Matching (8pts)

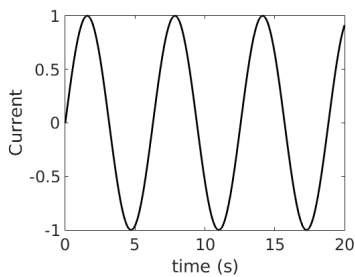


Throughout this problem, we assume $V_{in} = 1V$ for $t < 0$ and $0V$ for $t \geq 0$.

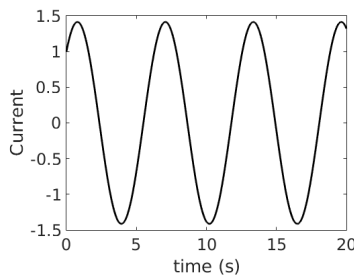
For this problem you are asked to **match** the transient behavior for **the current, i** , of the RLC circuit for various values of R , L , and C .

Circle your answer. There is no need to give any justification. However, 0 points will be awarded for an incorrect answer, 0.5 point will be awarded for leaving it blank and 4 points will be awarded for the correct answer

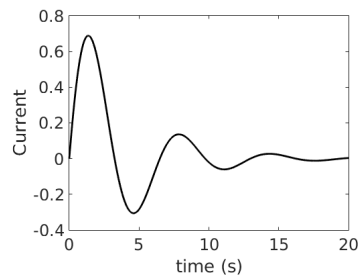
(a) (4 pts) For $R = 0\Omega$, $L = 1H$, $C = 1F$ Which one is the correct transient response of the current in the circuit?



(A)



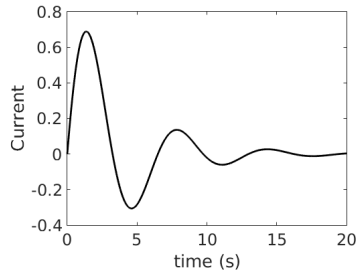
(B)



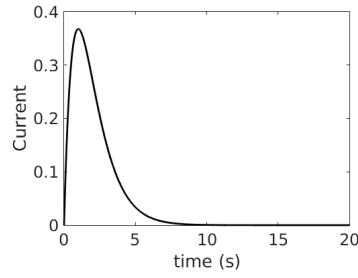
(C)

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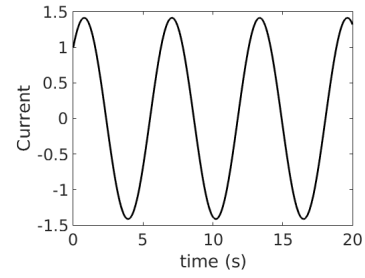
- (b) (4 pts) For $R = 0.5\Omega$, $L = 1H$, $C = 1F$ Which one is the correct transient response of the current in the circuit?



(A)



(B)



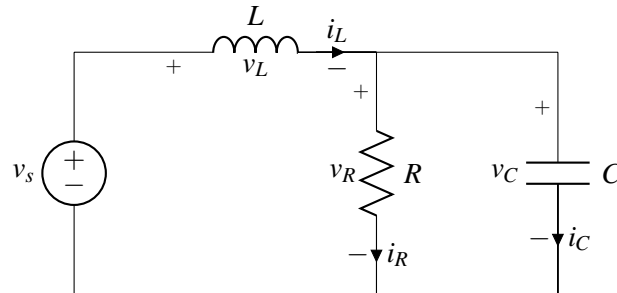
(C)

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Section 2: In The Zone (59 points)

8. RLC Problem (26 pts)

Consider the circuit below: let's try to analyze it with everything you know about circuits.



- (a) (3 pts) Assume $v_s = V_0$ for $t < 0$, and $v_s = 0$ for $t \geq 0$. **What is $v_C(0)$? What is $i_L(0)$?**
- (b) (3 pts) If $v_s = 0$ (a constant) for any $t \geq 0$, **what is the steady state value of v_C ?** (i.e. $v_C(t \rightarrow \infty)$)
What is the steady state value of i_L ?
- (c) (3 pts) **Write down the KCL equation on a node connecting the three passive components in terms of i_L , i_C and i_R .**
- (d) (3 pts) **Write down a KVL equation for the loop containing the voltage source, inductor and the capacitor in terms of v_s , v_L and v_C .**

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- (e) (6 pts) **Write down differential equations for v_C and i_L** using the relationships between the voltage across each component and the current through it, in addition to the equations obtained above. **Convert them into the following matrix form** (notice that $v_s = 0$ for any $t \geq 0$):

$$\begin{pmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{pmatrix} = A \begin{pmatrix} i_L \\ v_C \end{pmatrix}$$

- (f) (8 pts) For the differential equations above, we know the solution can be obtained from the general solutions $c_1 e^{\lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2$. **What are the values of λ_1 and λ_2 ? Express them in terms of R , L , C and constants.**

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9. Hold me and linearize me (13 pts)

Consider a non-linear two-dimensional system with states x_0 and x_1 and scalar input u that evolves according to the following coupled differential equations

$$\begin{aligned}\frac{d}{dt}x_0(t) &= \dot{x}_0 = x_1(t) \\ \frac{d}{dt}x_1(t) &= \dot{x}_1 = 4 - \left(\frac{u(t)}{x_0(t)}\right)^2\end{aligned}\tag{1}$$

- (a) (5 pts) **Find an input u_e so that if the system starts in state vector $\vec{x}_e = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and we apply the input $u(t) = u_e$, the system will always stay in that same state.**

- (b) (8 pts) **Write linearized state-space equations around \vec{x}_e and u_e . Convert them into the following form and find the matrices A and B .**

$$\frac{d}{dt}\vec{x}(t) = A(\vec{x} - \vec{x}_e) + B(u(t) - u_e)$$

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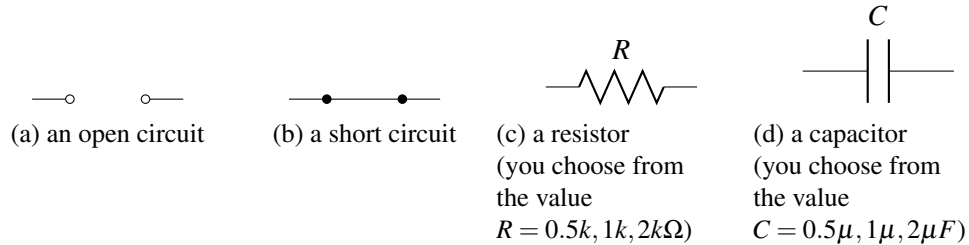
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10. Circuit Design (8 pts)

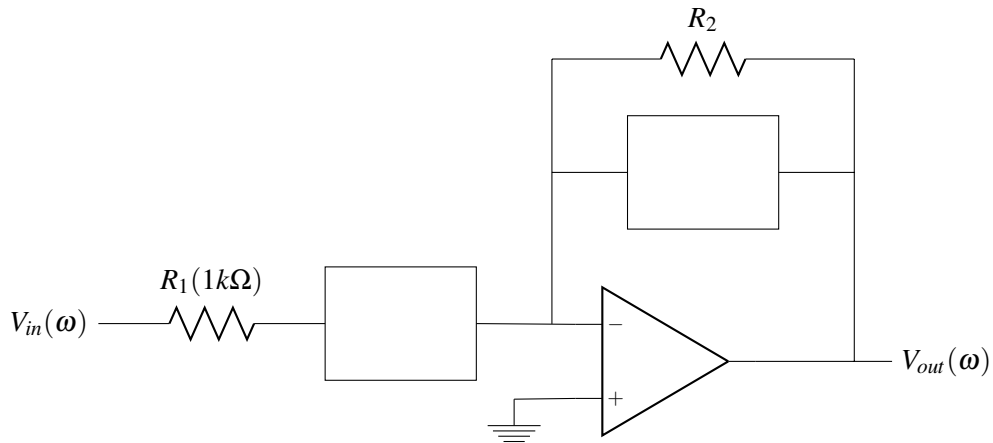
In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

Assume the op-amp is *ideal*.

You have at your disposal *only one of each* of the following components:



Consider the circuit below. The voltage source $v_{in}(t)$ has the form $v_{in}(t) = v_0 \cos(\omega t + \phi)$. The labeled voltages $V_{in}(\omega)$ and $V_{out}(\omega)$ are the phasor representation of $v_{in}(t)$ and $v_{out}(t)$. The transfer function $H(\omega)$ is defined as $H(\omega) = V_{out}(\omega)/V_{in}(\omega)$.



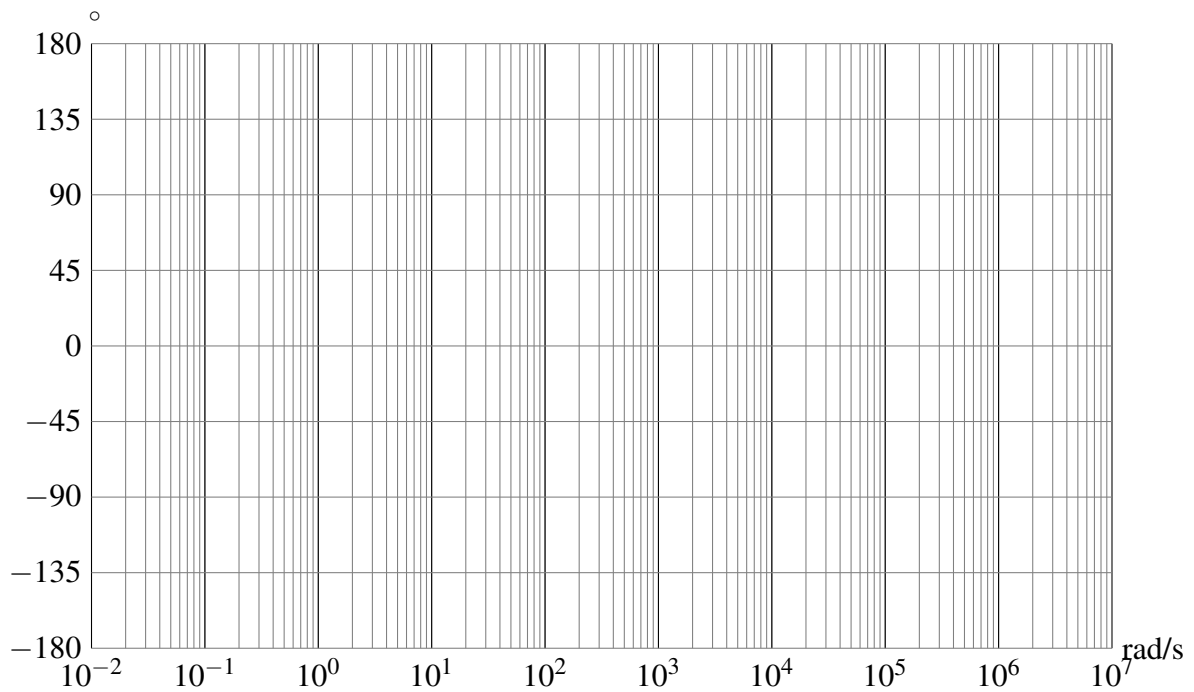
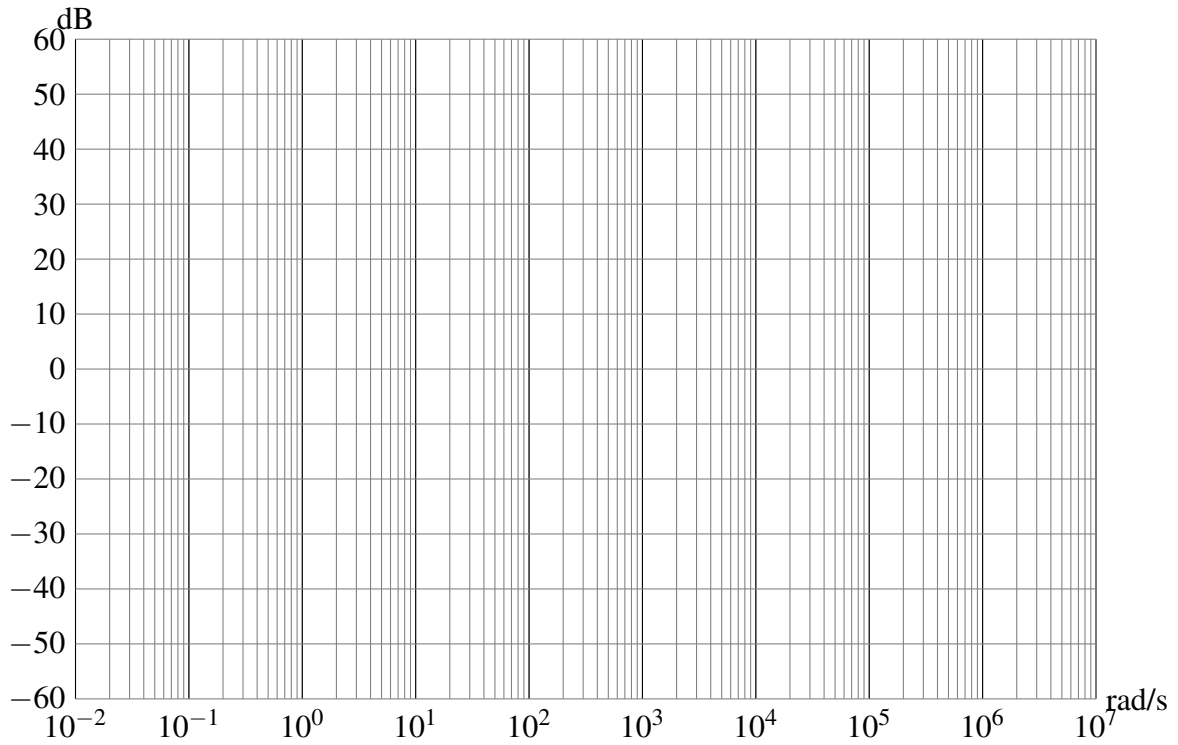
Let R_1 be $1k\Omega$. **Fill in the boxes and determine the value of R_2** so that

- It is a high-pass filter.
- $|H(\infty)| = 2$.
- $|H(10^3)| = \sqrt{2}$.

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11. Bode plot (12 pts)

Draw the Bode plot for the transfer function $H(\omega) = \frac{(j\omega \times 10)(10 + j\omega \times 10^{-3})}{(100 + j\omega \times 10)}$ Remember you have the Bode plot table in the next page!



Factor	Bode Magnitude	Bode Phase
Constant K	$20 \log K$ 0 dB 	$\pm 180^\circ$ if $K < 0$ 0° if $K > 0$
Zero @ Origin $(j\omega)^N$		$(90N)^\circ$
Pole @ Origin $(j\omega)^{-N}$		0° $(-90N)^\circ$
Simple Zero $(1 + j\omega/\omega_c)^N$		
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$		
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$		
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$		

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.]