Midterm 2

PROBLEM	POINTS	MAX
1		30
2		24
3		15
4		45
5		36

Personally, I liked the University; they gave us money and facilities, we didn't have to produce anything. You've never been out of college. You don't know what it's like out there. I've worked in the private sector--they expect results.
-Ghostbusters

Problem 1 30 points

a) Folded beams are often used to build MEMS sensors and actuators. Name two reasons why?

2 points

Two out of:

1- They can accommodate intrinsic stress better

2- The have a large stiffness in the longitudinal direction of the support beams

3- They occupy a smaller area.

b) In piezoresistive pressure sensors with a square diaphragm, four piezoresistors are usually used along the four sides of the square. Briefly explain why this is done instead of using two resistors along only two sides.

3 points

Four resistors are used to form the popular Wheatsone bridge. All four resistors are parallel to one another, two are parallel with two of the diaphragm sides, and the other two are perpendicular to the other two diaphragm sides. This is done so that one pair of resistors undergoes longiudinal stress and the other transverse stress. If these are placed on a (100) plane along the <110> directions the resistance change in the two pairs will have almost eual magnitude but opposite signs.

c) The ratio of stress to strain in a microstructure is given by (circle all that apply):

2 points

a) shear modulus	b) potential energy
c) momentum	d) Poisson's ratio
$\boxed{\text{e) Elastic modulus}}$	f) Bulk modulus

d) What is the Poisson's ratio, briefly explain?

2 points

Poisson's ratio relates the strain along a primary axis to that along a secondary axis.

e) Varying gap parallel-plate capacitors are often used for sensing applications. Name three shortcomings of this sensing mechanism?

5 points

1- Nonlinear

2- Susceptible to pull-in if the readout voltage becomes too large

3- Limited full-scale range for high-sensitivity devices

f) As discussed in class, in order to support mechanical structures we use several different beam designs, including a cantilever beam, a bridge structure, folded beams, and crab leg beams. The crab leg support is sometimes used because (for the following assume the same beam length, width, and thickness for all designs): **4 points**

a) It is more flexible than a bridge structure

b) It occupies less area than a bridge structure

c) It can accommodate a larger intrinsic stress than a bridge structure

d) It is less prone to stiction than a bridge structure

g) Vertical comb actuators have a larger force for a given device area and gap distance compared with lateral comb actuators. Briefly explain when this is the case.

4 points

The main reason is that the total perimeter area of vertical comb fingers where the force is generated is much larger than that of lateral combs. Therefore, for a given device area the force is much larger.

h) Name three advantages of capacitive pressure sensors over piezoresistive pressure sensors.

5 points

1- Higher normalized sensitivity
2- Lower power dissipation
3- Lower temperature sensitivity

i) Capacitive transducers are referred to as "energy conserving" transducers, whereas piezoresistive transducers are not. Briefly explain why.

3 points

The energy applied to capacitive transducers is stored in the capacitor as potential energy and is not dissipated. In ppiezoresistive transducers the energy is used up in the piezoresistors, and so these are not "energy conserving" transducers.

Freddy Shoop:	*They're as smart as you and me.*
Phil Gills:	*You and **I.***
Freddy Shoop:	*All of us.*

-Summer School

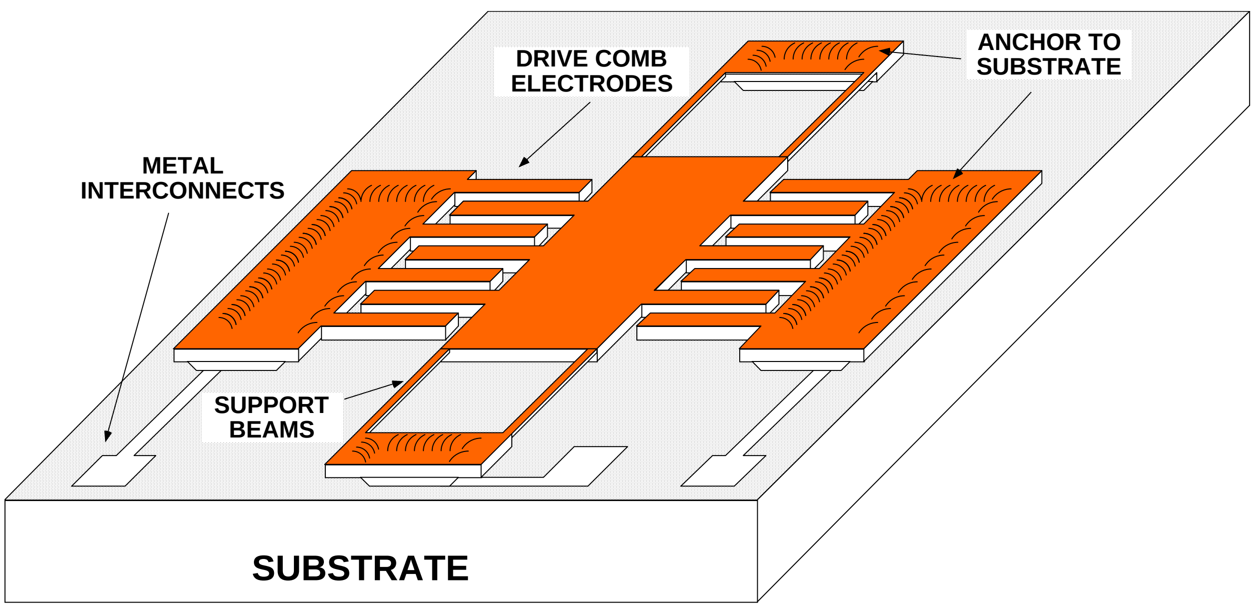
Problem 3 — 15 points

The following figure shows the basic structure of a comb-driven actuator. Note that the device is supported by four beams in a bridge-type configuration. Make the following assumption for the rest of this problem:

1. Silicon material: moderately-doped silicon
2. Thickness: 10µm in all areas
3. Gap between fingers: 2µm
4. Finger Width: 5µm
5. Distance between mass and substrate: 10µm

Now assume that the actuator is designed with the following dimensions:

- Each support beam is 200µm long and 3µm wide (note there are 4 beams altogether);
- The shuttle mass (the thick region in the middle) is 100µm wide (i.e. the distance between the two support beams is 100µm), and it is 1mm long;
- There are comb fingers along the entire length of the shuttle mass;
- The actuator is actuated by applying a DC voltage of 50V across one set of fingers;



a) Calculate the amount of force generated by the actuator to move the shuttle mass.

There are about 1000/(5+5+2+2)=72=n fingers

$$F = \frac{n\varepsilon h V^2}{g} \approx 8\mu N$$

b) Calculate the deflection of the device.

$$k = 4 \times \frac{Ewt^3}{l^3} = 23N/m$$

$$F = kx \Rightarrow x = \frac{8 \times 10^{-6}}{23} = 0.35\mu m$$

c) In order to increase the maximum deflection of the device without increasing the thickness of the silicon, what design changes will you make? Provide short answers and explain briefly.

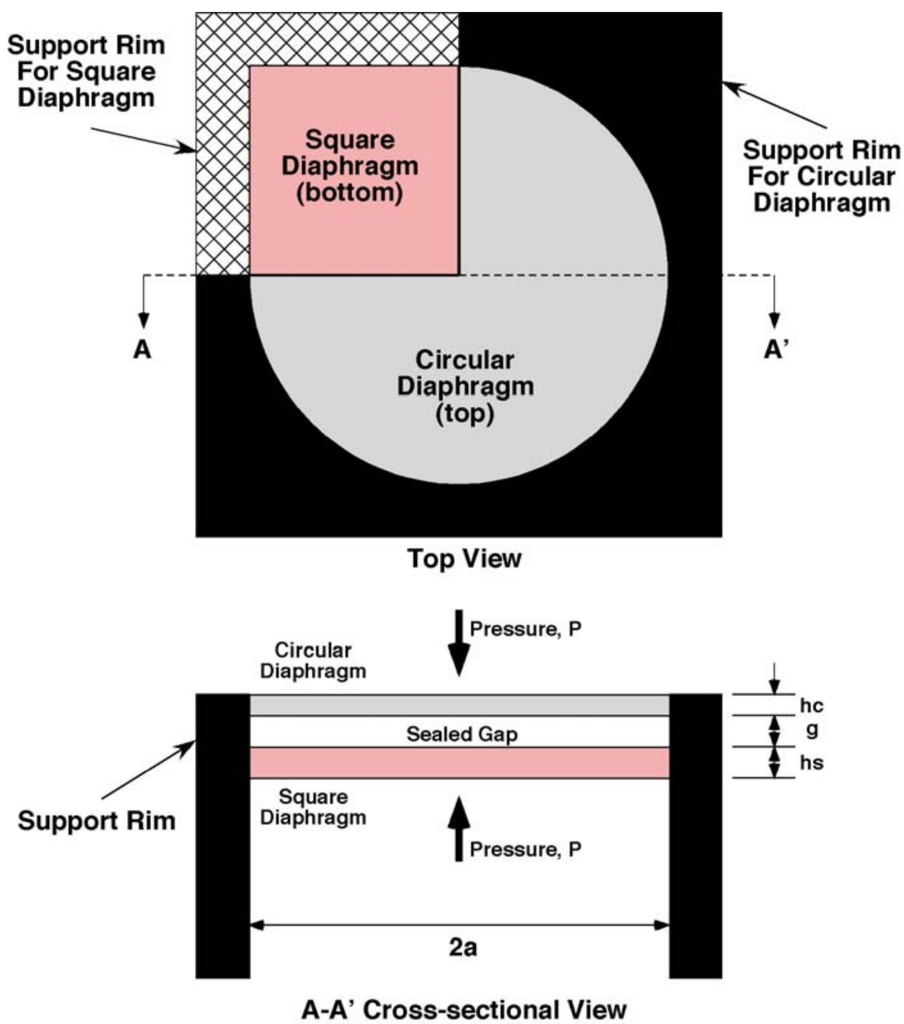
*To increase the deflection one needs to reduce the spring constant. This can be done by making the beams longer, or by using a folded beam structure. It is also possible to increase the force, which can be done by reducing the gap between fingers, or by increasing the number of fingers. (Increasing the voltage is not an acceptable answer since I asked for **design** changes).*

Prince Humperdinck: *Surrender!*
Westley: *You mean you wish to surrender to me? Very well, I accept.*

-The Princess Bride

Problem 4 45 points

This problem deals with the capacitive pressure sensor shown in the figure on next page. This sensor has a special structure. Both the top and the cross-sectional view (A-A') are shown in the figure. The sensor has two diaphragms, placed on top of each other as shown. The top diaphragm has a circular shape with a diameter of **2a** and thickness **hc**. The bottom diaphragm has a square shape with a side length of **2a** and thickness **hs**. The two diaphragms are separated by an air gap **g** as shown. Note that the air gap in between the two diaphragms is sealed and is not exposed to the external environment. The external environment has a pressure P above the pressure of the sealed gap, which is equally applied to both diaphragms. Both diaphragms are supported by an anchor as shown. Note that the top view of the structure (the drawing in the top of the figure) shows the diaphragms, and their support anchor. This picture also shows a portion of the top diaphragm removed in the upper left-hand corner to reveal the square diaphragm and its support anchor located below the circular diaphragm.



Part a **15 points**

We want to calculate the pressure sensitivity of this capacitive pressure sensor. That is we want to find an expression for the change in capacitance, **ΔC**, between the two diaphragms (note that **C** is not shown here), as a function of change in pressure **ΔP**.

Find the pressure sensitivity **ΔC/ΔP** as a function of device parameters given in the figure and the material properties of silicon.

Note that for this problem you can assume that there is no stress in either diaphragm, that the Poisson's ratio **ν** is zero, and that one can assume the deflections of the diaphragms can be assumed small compared with the gap **g**.

Please show all your work, and if you make any approximations please state them. For both diaphragms you can assume that the relationship between deflection, **y**, and Pressure difference, **P**, difference is given by:

$$y = \frac{3a^4 P}{16Eh^3}$$

Part a

We can write the capacitance of the device as a function of pressure as follows:

$$C = \frac{\varepsilon A_c}{(g - Total\ deflection)}$$

where A_c is the area of the circle (since the circle has a smaller area. Now we need to find the total deflection. This is eqaul to the sum of the deflections from the square and the circular diaphragms. The deflection vs. pressure was given and so we have:

$$P = \frac{16Eh^3}{3a^4}y$$

Of course this is different for the circular and the square diaphragms:

$$y_c = \frac{3a^4}{16Eh_c^{\ 3}}P = B_cP \quad for\ \ the\ \ circular\ \ diaphragm,\ and:$$

$$y_s = \frac{3a^4}{16Eh_s^{\ 3}}P = B_sP \quad for\ \ the\ \ square\ \ diaphragm.\ The\ B\ values\ are$$

simply constants used for convenience. The total deflection is:

$$y_t = y_c + y_s = B_cP + B_sP$$

Now, the capacitance is given as:

$$C = \frac{\varepsilon A_c}{g - y_t} = \frac{\varepsilon A_c}{(g - (B_c + B_s)P)}$$

We can now find the pressure sensitivity:

$$\frac{\Delta C}{\Delta P} = \frac{\varepsilon A_c(B_c + B_s)}{(g - (B_c + B_s)P)^2} \approx \frac{\varepsilon A_c(B_c + B_s)}{g^2}$$

Note that this is larger the sensitivity obtained with a single diaphragm, and if the two diaphragms have the same thickness the sensitivity is almost doubled.

Part b **10 points**

Find the effective spring constants of the circular (**kc**) and square (**ks**) diaphragms?

$F = kx = PA$ (where A is the area), and x is displacement.

For the two diaphragms, we have:

$$k_c = \frac{PA_c}{y_c} = \frac{16Eh_c^{\ 3}.(\pi a^2)}{3a^4} = \frac{16\pi Eh_c^{\ 3}}{3a^2}$$

$$k_s = \frac{PA_s}{y_s} = \frac{16Eh_s^{\ 3}.(4a^2)}{3a^4} = \frac{64Eh_s^{\ 3}}{3a^2}$$

Part c **20 points**

Now assume that a voltage **V** is applied between the two diaphragms. Derive an *approximate expression* for the pull-in voltage (This is the maximum voltage that can be applied between the two diaphragms before they will collapse) of this structure as a function of the spring constants of the two diaphragms.

$$F = \frac{\varepsilon A_c V^2}{2(g - y_t)^2} \quad ....(1)$$

We use the area of the circle here because it is smaller of the two.

But the total deflection was calculated before as the sum of the deflection of each diaphragm. For the two diaphragms, we have :

$$y_t = y_c + y_s = \frac{F}{k_c} + \frac{F}{k_s} = F\left(\frac{1}{k_c} + \frac{1}{k_s}\right) = \frac{F}{k_e}$$

Where k_e is the effective spring constant and is equal to :

$$\frac{1}{k_e} = \frac{1}{k_c} + \frac{1}{k_s}$$

So we can rewrite equation (1) as follows :

$$F = \frac{\varepsilon A_c V^2}{2(g - y_t)^2} = k_e\, y_t \quad \text{So we can rewrite this expression as :}$$

$$y_t = \frac{\varepsilon A_c V^2}{2k_e(g - y_t)^2}; \quad \text{or} \quad V = \sqrt{\frac{2k_e y_t}{\varepsilon A_c}}(g - y_t)$$

To find the pull - in voltage, take the derivative of V wrt. y_t, set it equal to 0 and then find the value for gap when this happens. We have done this before and we found that the maximu mdeflection was equal to 1/3 of the gap :

$$\text{Max } y_t = \frac{1}{3} g; \quad \text{At this gap we had found the pull - in voltage to be :}$$

$$V_{PI} = \sqrt{\frac{8k_e g^3}{27\,\varepsilon A_c}}$$

Note that this expression is not any different than that found for a beam, but the main difference is that the spring constant value is different and is equal to the effective spring constant. This effective spring constant is smaller than the spring constant of either diaphragm.

It's been swell, but the swelling's gone down.

-Tank Girl

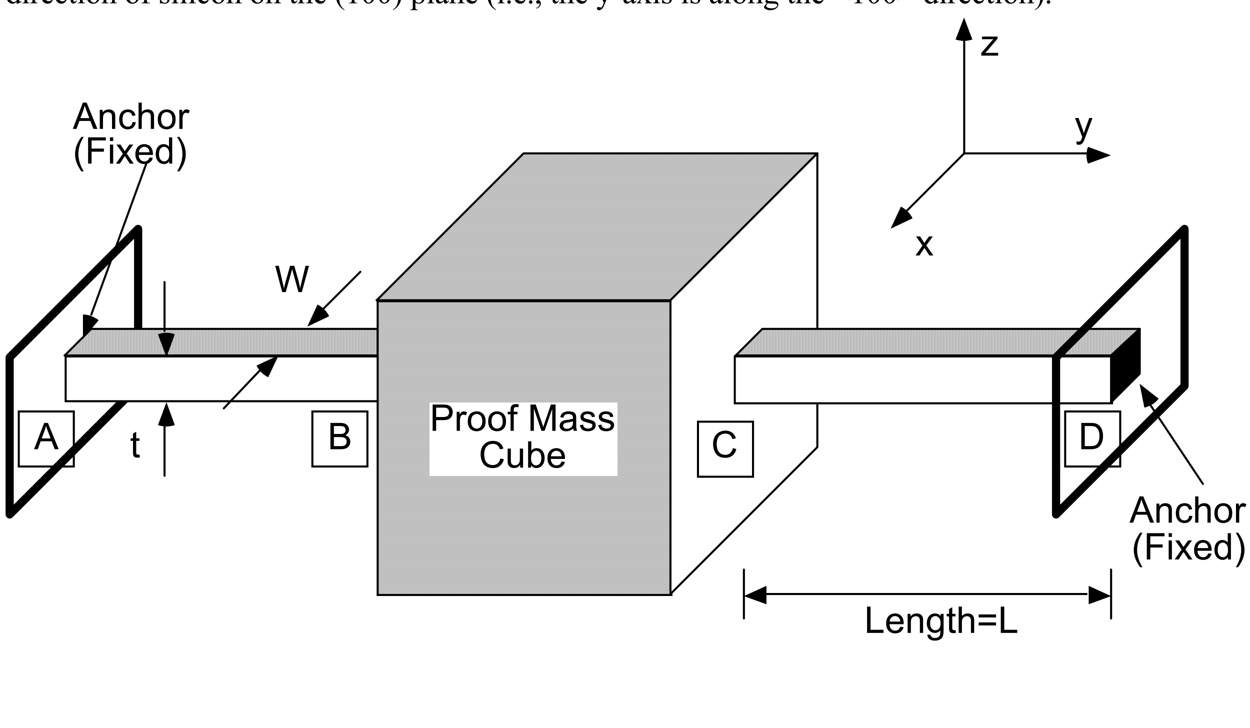
Problem 5

This problem deals with the piezoresistive accelerometer structure shown below. The accelerometer consists of a cube of silicon (side dimension of 1mm) that is supported using two beams as shown. Note that the entire structure is made of n-type silicon with a resistivity of 11.7 Ω-cm. Also, assume that the top surface of the mass cube (i.e., the xy plane) is the (100) plane of silicon. For the rest of this problem you can assume that you can measure the resistance of either of the two beams (i.e., section A-B, or section C-D), independently.

Assume the following dimensions for the accelerometer:

- The side of the proof mass cube: 1mm
- Length of each beam, L = 10μm
- Width of each beam, w = 1μm
- Thickness of each beam, t = 1μm
- Density of Si = 2330 Kg/m^3
- Young's Modulus of Si : 190 GPa (all directions)

For parts (a), (b), and (c) assume that the support beams are oriented along the <100> direction of silicon on the (100) plane (i.e., the y-axis is along the <100> direction).



Part a **9 points**

Calculate the acceleration sensitivity ($\Delta R/\Delta a$, and assume that R is the resistance of one of the beams A-B, or C-D) for this accelerometer if an acceleration along the x-axis is applied to the device.

Since the beam is under tension on one surface and under compression on the other, there will be no net change in resistance, so the acceleration sensitivity along the x-axis is zero.

Part b **9 points**

Calculate the acceleration sensitivity ($\Delta R/\Delta a$, and assume that R is the resistance of one of the beams A-B, or C-D) for this accelerometer if an acceleration along the z-axis is applied to the device.

Same as part (b), the total resistance change is always zero, so sensitivity is zero.

Part c: **9 points**

Calculate the acceleration sensitivity ($\Delta R/\Delta a$, and assume that R is the resistance of one of the beams A-B, or C-D) for this accelerometer if an acceleration along the y-axis is applied to the device.

A the mass moves in the y direction, one of the beams is axially compressed and the other is stretched. Calculate the axial force, from that calculate the stress, and then calculate the resistance change. For this part we are assuming that the beams are aligned to the <100> direction, so the piezoresistance coefficient of interest is π_{11}. For n-type silicon this is equal to -102.2×10^{-11} per Pa.

Note that the stress in this case is only axial, since there is only an axial load applied. As acceleration is applied in the y direction, one of the beams undergoes compression, and the other one tension, so the resistance in one increases, while the resistance in the other decreases.

Here are the calculations:

$$F = ma$$

$$m = 10^{-9} \times 2330 = 2.33 \times 10^{-6}$$

$$Stress = \sigma = \frac{F}{Beam\ \ cross-section\ \ Area} = \frac{F}{A}$$

$$\frac{\Delta R}{R} = \pi_l.\sigma = \pi_l \frac{F}{Area} \Rightarrow \frac{\Delta R}{\Delta a} = \pi_l \frac{mR}{Area}$$

$$But: \quad Resistance = R = \rho \frac{L}{A}$$

$$\frac{\Delta R}{\Delta a} = \pi_l \frac{m\rho L}{A^2}$$

Since the beam is aligned to the $<100>$ *direction, the longitudinal piezoresistance coefficient is simply* π_{11}*, and this is given above. So:*

$$\frac{\Delta R}{\Delta a} = \pi_l \frac{m\rho L}{A^2} = 102.2 \times 10^{-11}. \frac{2.33 \times 10^{-6} \times 11.7 \times 0.01 \times 10 \times 10^{-6}}{10^{-24}}$$

$$\frac{\Delta R}{\Delta a} = 2786 \quad \Omega / m / sec^2$$

$$Note\ \ that: \quad R_0 = 1.17 \ \ M\Omega, \ \ so:$$

$$\frac{\Delta R}{R.\Delta a} = 0.24 \ \ \% / m / sec^2$$

Part d **9 points**

Now assume that the support beams are oriented along the <110> direction of silicon. Calculate the acceleration sensitivity $\Delta R/\Delta a$ for acceleration along the y direction.

Note that you are supposed to obtain a numerical answer for each part. All the information you need, such as relevant piezoresistor coefficients, etc., can be obtained from various sources you have available, like your textbook, etc. **Make sure that you show your work.**

When the beams are aligned along the <110> direction, one has to use the longitudinal piezoresistance coefficient of silicon for that direction, which is a function of all of the primary piezoresistance coefficients, and so the new sensitivity is calculated as:

$$\frac{\Delta R}{\Delta a} = \pi_l \frac{m\rho L}{A^2}$$

$$\pi_l = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$$

$$For\ \ n-type\ \ Si:$$

$$\pi_{11} = -102.2 \times 10^{-11}$$

$$\pi_{12} = +53.4 \times 10^{-11}$$

$$\pi_{44} = -13.6 \times 10^{-11}$$

$$\pi_l = -62.4 \times 10^{-11}$$

$$\frac{\Delta R}{\Delta a} = 1701\ \ \Omega/m/sec^2$$

$$Note\ \ that:\ \ R_0 = 1.17\ \ M\Omega,\ \ so:$$

$$\frac{\Delta R}{R.\Delta a} = 0.15\ \ \%/m/sec^2$$