1a Given a set of measurements $a_i$, they are first summed and divided by $N$ to get the average, then the rms deviation from the average is calculated.

$$\bar{a} = \frac{1}{N} \sum_{i=1}^{N} a_i \quad \sigma_a = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (a_i - \bar{a})^2}$$

The standard deviation of the mean is the rms divided by the square root of $N$.

$$\sigma_{\bar{a}} = \frac{\sigma_a}{\sqrt{N}}$$

[1 point off for not defining $\bar{a}$]

1b Given two sets of measurements, compute the averages and the standard deviations of their means. Compute Student's $t$, which is the ratio of the difference of the averages divided by the standard deviation of that difference.

$$t = \frac{\Delta}{\sigma_\Delta} = \frac{\bar{a} - \bar{b}}{\sqrt{\sigma^2_a + \sigma^2_b}} = \frac{\bar{a} - \bar{b}}{\sqrt{\sigma^2_a / N + \sigma^2_b / N}}$$

Look up the probability of exceeding $|t|$ from the Student's $t$ table for that value of $t$ and $2N-2$ degrees of freedom.

[6 points off for not computing Student's $t$]
[3 points off for not using the Student's $t$ probability table]
[1 point off for not mentioning the number of degrees of freedom]

2a If an arbitrary waveform is periodic with period $P$, then the Fourier transform has non zero values only at integer multiples of the frequency $1/P$.

[5 points off if the answer was only “discrete in frequency” since you could say more]
[3 points off if only one frequency $f = 1/P$ was given]
[2 points off if harmonics were mentioned but the spacing $\Delta f = 1/P$ was not given]

2b If an arbitrary waveform $h(t)$ is multiplied by a window function $w(t)$, then the Fourier transform of the result $g(t) = h(t) w(t)$ is the convolution of the Fourier transforms of $h(t)$ and $w(t)$.

3a Sampling at 1024 Hz for $S = 0.5$ seconds produces 512 samples. The FFT produces 512 coefficients, and the magnitude $F_k$ of the $k$th coefficient corresponds to a frequency of $k/S = 2k$ Hz.
F_0 = 0
F_5 and F_{512-5} are the only non-zero Fourier coefficient
No aliasing, since 10 Hz < F_s/2 = 512 Hz
No spectral leakage, since exactly 5 cycles are sampled in 0.5 sec
[2 points off if harmonics of 10 Hz are shown]
[2 points off if frequency scale in Hz not shown in at least one of 3a, 3b, 3c, or 3d]

3b

F_5 and F_6 are non-zero
Spectral leakage, because 5.5 cycles are sampled in 0.5 sec
Values around these have spectral leakage, falling off as 1/f
No aliasing, since 11 Hz < F_s/2 = 512 Hz
To fix spectral leakage, multiply the time samples by a Hanning window and/or increase the sampling window S.

3c
250 cycles are sampled in 0.5 sec - no spectral leakage
$H_{250}$ and $H_{512-250}$ ($H_{250}$ and $H_{262}$) are non-zero
No aliasing because 500 Hz is below the Nyquist frequency of $1024 \text{ Hz}/2 = 512 \text{ Hz}$
No spectral leakage because exactly 250 cycles are sampled

3d

262 cycles are sampled in 0.5 sec - no spectral leakage
$H_{262}$ and $H_{512-262}$ ($H_{250}$ and $H_{262}$) are non-zero
No aliasing because exactly 262 cycles are sampled
Aliasing occurs because 524 Hz is above the Nyquist frequency of $1024 \text{ Hz}/2 = 512 \text{ Hz}$
Fix aliasing by sampling higher than $2 \times 524 = 1048 \text{ Hz}$.
Fourier magnitudes for 524 Hz look exactly like Fourier magnitudes for 500 Hz

3e See above for answers
[2 points off for each incorrect answer to spectral leakage in 3a, 3b, 3c, or 3d]
[2 points off for each incorrect answer to aliasing in 3a, 3b, 3c, or 3d]
[2 points off each for not fixing spectral leakage in 3b or fixing aliasing in 3d]

4a

The following are essential [3 points off for each omitted]:
• Connect 16 bits of the parallel output port to the input of the 16 bit D/A converter
• Connect the analog output of the D/A to the analog inputs of all A/Ds
• Connect the 12 bit A/Ds to separate tri-state drivers
• Connect the outputs of the tri-state drivers together to form a data bus
• Connect the data bus to 12 bits of the parallel input port
• Provide 8 separate output port lines for initiating conversion of the 8 A/Ds
• Provide 8 separate output port lines for enabling the 8 separate tri-state drivers
• Provide input for 8 separate input port lines to indicate when individual A/Ds have completed conversion

(OK to connect the start conversion and output enable signals for each A/D signal path, provided the signal timing was properly designed)

---

**4b** The steps needed to measure the first transition voltage \( V(0,1) \) for the first A/D converter.

1. Set \( N=0 \)
2. Put \( N \) on D/A
3. wait 10\( \mu \)s until D/A has settled [using \( \text{wait}(10) \)]
4. Put low-high edge on output line to start conversion
5. Wait until output data available
6. enable tri-state 1 (disable all others)
7. read input port to get value \( M \)
8. Put high-low edge on output port to end conversion cycle
9. if \( M=0 \), increase \( N \) by one and loop back to step 2
10. If \( M=1 \), save (D/A voltage step)(N-1/2) as the transition voltage

---

**4c** Send successive 16-bit numbers 0 to \( 2^{16} - 1 \) to the D/A converter and convert the analog output with the A/D. Whenever the A/D output value changes, store the corresponding D/A value in a transition voltage table.

To determine **absolute accuracy**, compare the D/A values for each A/D output transition with the ideal transition values. Since the step size of the D/A is 16 times finer than the average step size of the A/D, this design can measure the A/D transition voltages to an accuracy of 1/16 LSB.

[3 points off if only 12 D/A bits are varied. The transition voltages (or the center of the steps) cannot be determined accurately unless more than 12 bits of the accuracy of the 16-bit D/A is used.]

**4d** Determine the D/A values corresponding to first and last A/D transition voltages, and the equation of the line that passes through them. **Linearity** is a measure of how closely the other measured transition values pass through the line.

**4e** Determine the difference in D/A values between each A/D transition voltage. The **differential linearity** is a measure of the equality of those differences. Alternatively, the A/D step sizes could be determined as the number of successive D/A inputs that produce the same A/D output.
4f. The method can determine the A/D accuracies to 1/16 LSB (±1/32 LSB was OK). Note that 1 A/D LSB = 16 D/A LSBs.

[5 points off for an answer of 1/2 LSB]

5a. A/D converter circuit

![A/D converter circuit diagram](image)

[?? points off for not providing a circuit that converts \(2^N\) comparator output to an \(N\)-bit address]

5b. With an input of 1.27 volts, the lower 127 comparators have an output of one and the upper comparators have an output of zero. This pattern of ones and zeros is converted into the number 127, which is a digital representation of the voltage.

When the input voltage is increased to 1.28 volts, the lowest comparator that previously had an output of zero now has an output of one, and this pattern is converted into the number 128.
145M Final Exam Grades (12 undergraduate and 2 graduate students):

<table>
<thead>
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<th>Problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
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<td>56.1</td>
<td>61.3</td>
<td>26.1</td>
<td>178.2</td>
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<tr>
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<td>3.3</td>
<td>4.3</td>
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<td>3.1</td>
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<tr>
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<td>20</td>
<td>60</td>
<td>70</td>
<td>30</td>
<td>200</td>
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The scores for each laboratory exercise report were averaged and it was found that labs 9 and 21 had unusually low average scores. A lab point bonus system was devised that (1) compensated for the differences in the average scores of the different lab exercises and (2) included an additive constant so that it never subtracted from any student’s numerical score.

145M Numerical Grades (12 undergraduates only):

<table>
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<tr>
<th></th>
<th>Lab total</th>
<th>Lab bonus</th>
<th>Lab Partic.</th>
<th>Midterm #1</th>
<th>Midterm #2</th>
<th>Final</th>
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<tr>
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<td>7.6</td>
<td>85.8</td>
<td>89.9</td>
<td>77.4</td>
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<td>rms</td>
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<td>6.6</td>
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<td>11.1</td>
<td>8.8</td>
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<td>24.7</td>
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<tr>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>1000</td>
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145M Letter Grade Distribution

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<thead>
<tr>
<th>Letter Grade</th>
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<tbody>
<tr>
<td>A+</td>
<td>944.5</td>
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<tr>
<td>A</td>
<td>923.0, 930.0, 935.5 (G)</td>
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<tr>
<td>A–</td>
<td>906.0, 906.8, 910.1, 915.0</td>
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<tr>
<td>B+</td>
<td>887.5, 887.7, 890.7</td>
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<tr>
<td>B</td>
<td>876.1</td>
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<tr>
<td>B–</td>
<td>none</td>
</tr>
<tr>
<td>C+</td>
<td>855.3</td>
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<tr>
<td>C</td>
<td>843.9 (G)</td>
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</tbody>
</table>

(G) = graduate student