Solutions for Midterm #2 - EECS 145M Spring 1997

1a The following are essential:

- Connect the parallel output port to the input of the D/A converter
- Connect the analog output of the D/A to the analog input of the A/D
- Connect the output of the A/D to the parallel input port
- Start A/D conversion under computer control (using the "Ready for input data" handshaking line)
- Use the "Data ready" A/D output to strobe the data onto the parallel input port
- Use the "Data ready" A/D output to signal the "Input data available" handshaking line



- **1b** 1 Set "Ready for input data" low, which makes "Input data available" low
 - 2 Start loop over all values of n from 0 to $2^{16} 1$
 - 3 Write n to the D/A converter
 - 4 Write a low, then a high to "Ready for input data" to start A/D conversion
 - 5 Read "Input data available" in a loop until it goes high
 - 6 When the A/D converter finishes, it strobes the data onto the input port and sets the "Input data available" line high
 - 7 The program detects this and reads the input port
 - 8 The program sets "Ready for input data" low, which causes the A/D converter to set "Input data available" low
 - 9 If the A/D output has changed from the last value read (say from m–1 to m), store the value of n, which corresponds to the $V_{m-1, m}$ transition voltage.
 - 10 Loop back to step 2
 - 11 Tabulate the difference between the measured transition voltages $V_{m-1, m}$ and the ideal transition voltages V(m-1,m) = (m-0.5)(4.095V/4095) = 0.001 V (m-0.5). The maximum value is the maximum absolute accuracy error.

[3 points off if only 12 D/A bits are varied. The transition voltages (or the center of the steps) cannot be determined accurately unless more than 12 bits of the accuracy of the 16-bit D/A is used.]

- [4 points off if the method is not automatic]
- [3 points off if the transition voltages are not measured]
- [3 points off if handshaking steps not indicated]

- 1c As part 1b above, but compare the measured transition voltages $V_{m-1, m}$ (as a function of m) with the straight line passing between the first measured transition voltage $V_{0,1}$ and the last measured transition voltage $V_{4094, 4095}$. The largest deviation is the maximum linearity error. [3 points off if the straight line is defined in terms of V_{ref}^- and V_{ref}^+ (which is required for the absolute accuracy error). The maximum linearity error requires a straight line that passed through the *measured* end points]
- 1d As part 1b above, but compare the A/D step sizes $V_{m, m-1} V_{m-1, m}$ with their average value. The largest deviation is the maximum differential linearity error. Alternatively, the A/D step sizes could be determined as the number of successive D/A inputs that produce the same A/D output.

Note: It was essential to use the concept of a "table of transition voltages" to answer parts \mathbf{b} and \mathbf{c} of this problem.

- 1e Since the D/A has an absolute accuracy of ± 1 LSB, and its step size is 16 times finer than the average step size of the A/D, this design can measure the A/D transition voltages to an accuracy of $\pm 1/16$ of the A/D LSB. As a result, the accuracy is 1 A/D LSB for the maximum absolute and linear errors and 1.414 A/D LSB for the maximum differential linearity error (difference between two random errors).
- **2a** The low-pass Butterworth anti-aliasing filter must preserve signal amplitudes to an accuracy of 1%, and reduce frequencies above one-half the sampling frequency by a factor of less than 0.2%. The highest frequency in the signal is the 15th harmonic of 2 kHz, or 30 kHz. The signal amplitude accuracy of 1% corresponds to a filter gain of 0.99 at 30 kHz.

If we choose an 8-stage Butterworth anti-aliasing filter (see the table at the end of the exam), the gain of 0.99 occurs at $f_1 = 0.784$ f_c and the gain of 0.001 (a conservative value to reduce the higher harmonics below 0.2%) occurs at $f_2 = 2.317$ f_c. Since $f_1 = 30$ kHz, $f_2 = (2.371/0.784)(30$ kHz) = 91 kHz.

If we choose a 12-stage Butterworth anti-aliasing filter, the gain of 0.99 occurs at $f_1 = 0.850 f_c$, the gain of 0.001 occurs at $f_2 = 1.778 f_c$, and $f_2 = (1.778/0.850)(30 \text{ kHz}) = 63 \text{ kHz}$.

[8 points off if 60 kHz sampling alone is used to avoid aliasing- it is essential to limit the highest frequencies in the signal with a low pass filter *before* sampling]

[5 points off for not recognizing that the signal we want extends to 30 kHz]

[2 points off for not checking that filter gain is >99% at 30 kHz and <0.2% at $f_s/2$]

2b If we choose the 8-stage Butterworth anti-aliasing filter described in 2a, frequencies above f₂ = 91 kHz are eliminated, and the minimum sampling frequency is 182 kHz.
If we choose the 12-stage Butterworth anti-aliasing filter described in 2a, frequencies above f₂

= 63 kHz are eliminated, and the minimum sampling frequency is 126 kHz.

[5 points off for assuming that the LPF cut off abruptly at the corner frequency]

- **2c** Since the frequency resolution in the FFT was required to be f = 0.5 Hz, the minimum time needed to take the samples is S = 1/f = 2 s.
- **2d** If using an 8-stage Butterworth filter, the number of samples is (2 s)(182 kHz), or 364k samples. In practice, we would sample for 2.9 s to acquire 524,288 samples, a power of two. If using a 12-stage Butterworth filter, the number of samples is (2 s)(126 kHz), or 252k samples. In practice, we would sample for 2.1 s to acquire 262,144 samples, a power of two. [1 point off if time and/or f_s not adjusted to give a power of 2]
- **2e** A Hanning window is essential because it is impossible to ensure that a complete number of samples will be taken at all the frequencies where signals are present. Without the Hanning window to suppress spectral leakage, frequency components will be spread out over many frequencies with a 1/f falloff, and it will be impossible to meet the 0.2% accuracy requirement.

- **2f** For a sampling window from S = 2 to 3 s (depending on the anti-aliasing filter used), the Fourier coefficient H₁ corresponds to a frequency of f = 1/S, somewhere between 0.35 and 0.5 Hz
- **2g** The ith harmonic will occur at a frequency of 500i Hz, and (assuming a sampling window of 2 s), at the Fourier coefficient H_{1000i} . The Hanning window will spread the non-zero amplitudes over several Fourier coefficients near H_{1000i} , but will also reduce the spectral leakage to a small value. Above 30 kHz ($H_{60,000}$), the harmonic amplitudes will drop rapidly due to the low-pass anti-aliasing filter. The right half of the plot of FFT magnitudes vs. Fourier index will be a mirror image of the left half. (Assuming 262,144 samples, $H_{262,144-1000i}$ will be the complex conjugate of H_{1000i} .). If there is a d.c. component, H_0 will be non-zero.

[5 points off for first harmonic only]

[2 points off for not mentioning that upper half of FFT magnitude plot is mirror image of lower half]

Midterm #2 class statistics:

Problem	max	average	rms
1	50	39.2	6.2
2	50	36.3	7.4
total	100	75.5	12.2

Grade distribution:

Range	number	approximate
		letter grade
41-50	1	Ď
51-60	0	
61-70	3	B-
71-75	2	В
76-80	4	А
81-90	1	A+
91-100	2(G)	A+