PROBLEM 1 (50 points)

Design a computer controlled system for the automatic testing of 12-bit A/D converters.

You are provided with the following:
- A microcomputer equipped with a 16-bit parallel input port, and a 16-bit parallel output port.
- A 16-bit D/A converter with ±1 LSB absolute accuracy.

You may assume the following:
- The 16-bit parallel output port is in “transparent” mode. A 16-bit word $A$ written to the output port using the command `outport(1, A)` immediately appears on the output lines.
- The 16-bit parallel input port requires a low-to-high edge on a “strobe” input line for external data to be latched onto the 16 bit registers. The program can read these registers using the command `$B = \text{inport}(1)$`.
- The parallel input port has an “input data available” line that can be asserted high or low by an external device and read by the program using the command `C = \text{inport}(2)`.
- The parallel input port has an external “ready for input data” line that can be set high using the program command `outport(2, 1)`, and set low using `outport(2, 0)`.
- The A/D converter requires a “start conversion” low-to-high signal and after conversion provides a “data ready” low-to-high signal that goes low when “start conversion” goes low.
- The A/D reference voltages are $V_{\text{ref}^-} = 0.0000$ V and $V_{\text{ref}^+} = 4.095$ V
- The D/A reference voltages are $V_{\text{ref}^-} = 0.0000$ V and $V_{\text{ref}^+} = 4.096$ V
1a. [25 points] Draw a block diagram of the major components, including the A/D circuit being tested. Show and label all essential components, data lines, and control lines.

1b. [10 points] How would you measure the maximum absolute accuracy error of the A/D? (Explain the procedure in steps or with a flow diagram.)
1c. [5 points] How would you measure the maximum linearity error?

1d. [5 points] How would you measure the maximum differential linearity error?

1e. [5 points] With what accuracy could this system measure the quantities in parts b., c., and d. in units of 1 LSB of the A/D?

PROBLEM 2 (50 points)
Design a microcomputer-based system for using the FFT to analyze the harmonic content of musical instruments.

The design requirements are:
- The instruments have a fundamental frequency (first harmonic) ranging from 50 Hz to 2 kHz.
- The system must sample the waveform with an amplitude accuracy of ±1% over all frequencies of interest.
- The system must compute harmonic magnitudes from the 1st to the 15th harmonic with an accuracy that is 0.2% of the largest harmonic. (You may assume that at and above the 15th harmonic, the magnitudes decrease with increasing frequency.)
- Neighboring Fourier coefficients correspond to frequencies differing by 0.5 Hz.
2a. [10 points] How does your design avoid aliasing? Give details.

2b. [10 points] What is the minimum sampling frequency required?

2c. [5 points] What is the minimum time needed to take all the required samples?

2d. [5 points] What is the minimum number of samples required?
2e. [5 points] Would a Hanning window be useful in your design? Explain your reasoning.

2f. [5 points] To what frequency does the first FFT coefficient $H_1$ correspond?

2g. (10 points) For a musical instrument with a first harmonic frequency of 500 Hz, which FFT magnitudes would you expect to be non zero?

Equations, some of which you might find useful:

\[ G(a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2} \left(\frac{a-\mu}{\sigma}\right)^2\right] \quad \mu = \bar{a} = \frac{1}{m} \sum_{i=1}^{m} a_i \]

\[ \sigma_a^2 = \text{Var}(a) = \left(\frac{1}{m-1}\right) \sum_{i=1}^{m} R_i^2 = \left(\frac{1}{m-1}\right) \sum_{i=1}^{m} (a_i - \bar{a})^2 \quad \text{Var}(\bar{a}) = \text{Var}(a)/m \]

\[ t = \frac{\Delta}{\sigma_\Delta} = \frac{\bar{a} - \bar{b}}{\sqrt{\sigma_a^2 + \sigma_b^2}} = \frac{\bar{a} - \bar{b}}{\sqrt{\sigma_a^2/m_a + \sigma_b^2/m_b}} \quad \sigma_f^2 = \left(\frac{\partial f}{\partial a_1}\right)^2 \sigma_{a_1}^2 + \left(\frac{\partial f}{\partial a_2}\right)^2 \sigma_{a_2}^2 + \ldots + \left(\frac{\partial f}{\partial a_n}\right)^2 \sigma_{a_n}^2 \]

\[ f = k(a+b) \quad \sigma_f^2 = k^2 \left(\sigma_a^2 + \sigma_b^2\right) \quad f = kab \quad \sigma_f^2/k^2 = \sigma_a^2/a^2 + \sigma_b^2/b^2 \]
$$v = \frac{s - r q}{m s - r^2}$$ and $$b = \frac{m q - r t}{m s - r^2}$$ where $$r = \sum n_i$$, $$s = \sum n_i^2$$, $$q = \sum n_i V_i$$, $$t = \sum V_i$$

$$V(n) = V_{ref}^- + n \left( \frac{V_{ref}^- - \Delta V}{2^N} \right) = V_{min} + n \left( \frac{V_{max}^- - V_{min}^-}{2^N - 1} \right)$$

$$n = \left[ \frac{V - V_{ref}^-}{\Delta V} + \frac{1}{2} \right]$$

$$V(n - 1, n) = V_{ref}^- + (n - 0.5) \Delta V \quad \Delta V = \frac{V_{ref}^- - V_{ref}^-}{2^N - 1}$$

$$F_n = |H_n| = \sqrt{\text{Re}(H_n)^2 + \text{Im}(H_n)^2} \quad \tan \phi_n = \text{Im}(H_n)/\text{Re}(H_n)$$

For $$h_k = \sum_{i=0}^{M-1} a_i \cos(2 \pi k / M) + b_i \sin(2 \pi k / M)$$

$$f_{\text{max}} = f_s / 2 \quad \Delta t = 1 / f_s \quad S = M \Delta t \quad \Delta f = 1 / S \quad h(t) = 0.5 [1.0 - \cos(2 \pi t / S)]$$

$$y_i = A_1 x_{i-1} + A_2 x_{i-2} + \ldots + A_M x_{i-M} + B_1 y_{i-1} + \ldots + B_N y_{i-N}$$

If $$a(t) = \int b(t') c(t - t') dt' = b(t) * c(t)$$, then FFT(a) = FFT(b) multiplied by FFT(c)

$$f_{\text{max}} = \frac{1}{2 N + 1 / \pi T}$$

$$V(t) = V(0) e^{-t / \pi C}$$

$$\frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \frac{1}{\sqrt{1 + (f / f_c)^{2n}}}$$ (see table below)

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