1a The **transparent latch** input and output are digital while the **sample and hold amplifier** input and output are analog. Both have similar control lines whose state determines whether the output is equal to the input or held at its last value.  
[1 point off for describing both but not stating what is different]  
[4 points off for describing both without mentioning analog or digital]  
[4 points off for stating that the transparent latch is always transparent but the S/H can be either sample or hold]  

1b **D/A differential linearity error** is the difference between output voltage step sizes and the average step size. **D/A relative accuracy error** is the difference between output voltage (as a function of input number) and a straight line passing through the minimum and maximum values (end points).  
[5 points off for defining differential linearity error as relative accuracy error and defining relative accuracy error as absolute accuracy error]  

1c **Frequency aliasing** is caused by insufficient sampling and causes frequencies above one-half the sampling frequency to appear as lower frequencies. **Spectral leakage** is caused by sampling a non-integer number of cycles, which produces a discontinuity at the edges of the sampling window, which adds erroneous contributions to many Fourier coefficients. (More precisely, the observed Fourier transform is the true Fourier transform convolved with the Fourier transform of the sampling window)  

2a DS, SD  
2b TR, SA, HF, SD  
2c FL  
2d SA or SD  
2e SA, HF  

3a 1. increasing the sampling frequency \( f_s \) will increase the Nyquist limit \( f_{\text{max}} = f_s / 2 \), allowing the sampling of higher frequencies without aliasing  
2. adding more poles would help reject frequencies above one-half the sampling frequency  
3. increasing the corner frequency will be necessary so that the filter will not limit \( f_{\text{max}} \)  

3b 5. Increasing the sampling time \( S \) decreases the frequency resolution \( \Delta f = 1/S \).  

3c 4. Adding a Hanning window will reduce spectral leakage  
5. Increasing \( S \) will separate the two harmonic components for easier observation and will also further reduce the small spectral leakage of the Hanning window.  
6. Increasing the number of A/D bits is optional [no points off either way]
4a

[2 points off for not specifying the type of A/D converter]
[4 points off for not using the timer to control the conversion rate]

4b
1. Counter/timer generates an external pulse “start conversion”
2. The counter/timer pulse also puts the S/H in hold mode
3. A/D converter converts analog data
4. A/D converter asserts data on its output lines
5. A/D converter signals “data ready”
6. S/H goes back to sample mode
7. Program detects “data ready”
8. Program reads data

4c
A 12-stage Butterworth filter has a gain = 0.999 at $f_1/f_c = 0.772$. Since we want 0.1% accuracy for all frequencies at or below 18 kHz, $f_1 = 18$ kHz, and $f_c = 18$ kHz / 0.772 = 23.3 kHz.

4d
A 12-stage Butterworth filter has a gain = 0.001 at $f_2/f_c = 1.778$. With $f_c = 23.3$ kHz, $f_2 = (23.3$ kHz) (1.778) = 41.5 kHz. So the filter reduces all frequencies above 41.5 kHz below 0.1% of their amplitude, as desired, and we must sample at twice this frequency, or 83 kHz.

[4 points off for $f_s = 18$ kHz x 2] [4 points off for $f_s = 2 f_c$]

4e Without a Hanning window, the frequency resolution $\Delta f$ must be no more than 0.5 Hz to resolve equal harmonic components separated by 1 Hz. (If $\Delta f = 1$ Hz, it will be impossible to see a valley between two equal harmonics separated by 1 Hz.) With a Hanning window, however, neighboring Fourier coefficients can have as much as 1/2 the amplitude, so $\Delta f = 0.33$ Hz would be necessary to observe a valley between two equal harmonics separated by 1 Hz. So the sampling window must be at least 3 s long.

[4 points off for $S = 1$ s]
4f
A sampling rate of 83 kHz will accumulate 249,000 samples in 3 s. To make a power of two (262,144), sample for just a bit more than 3s.

4g
10 bits will provide a voltage resolution of ±0.5 LSB/1023 LSB = ±0.05%

5a
1. Connect the output of the D/A to the amplifier input and the output of the amplifier to the A/D input
2. Generate an impulse (maximum voltage, width $\Delta t$) at the input of the power amplifier, and immediately sample the output as a series of voltages $c_i$, with time spacing $\Delta t$.
3. Generate one cycle of the periodic waveform $a(t)$ as a series of values $a_i$, with a time spacing $\Delta t = t_{i+1} - t_i$ so that the sampling frequency $f_s = 1/\Delta t > 2f_{\text{max}}$.
4. If $b(t)$ is the amplifier input that produces the desired output $a(t)$, then

$$a(t) = b(t) \ast c(t)$$

$$F(a) = F(b) \ast F(c) = F(a) \ast F(d) \ast F(c)$$

$$F(d) = 1/F(c)$$

$$d(t) = F^{-1}\left[1/F(c)\right] = F^{-1}\left[\frac{C_r - jC_i}{C_r^2 + C_i^2}\right]$$

5b
1. Generate an impulse (maximum voltage, width $\Delta t$) at the input of the power amplifier, and immediately sample the output as a series of voltages $c_i$, with time spacing $\Delta t$ the same as the stored values $a_i$.
2. If $d_i = d(t_i)$ is the desired FIR filter, then

$$a(t) = [a(t) \ast d(t)] \ast c(t),$$

where $c(t)$ is the impulse response of the amplifier.

$$a(t) = [a(t) \ast d(t)] \ast c(t)$$

$$F(a) = F(a \ast d) \ast F(c) = F(a) \ast F(d) \ast F(c)$$

$$F(d) = 1/F(c)$$

$$d(t) = F^{-1}\left[1/F(c)\right] = F^{-1}\left[\frac{C_r - jC_i}{C_r^2 + C_i^2}\right]$$

3. Program a digital filter so that the desired values $a_i$ from the disk are continuously transformed into the amplifier input values $b_i$:

$$b_i = \sum_k d_{k-i} a_i$$

The amplifier will then convolve the values $b_i$ with its impulse response $c(t)$ to produce the sequence $a_i$ at its output, as desired.
**145M Final Exam Grades:**

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<th>3</th>
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* 10 undergraduates only; average of 3 graduate students was 174.0, rms 18.2

**145M Numerical Grades (9 passing undergraduates only):**

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**145M Letter Grade Distribution (passing students only)**

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(G) = graduate student