Solutions for Midterm #2 - EECS 145M Spring 2010

1.1 Frequency aliasing is caused by sampling at a frequency f_s that is less than twice the highest frequency f_{max} in the waveform.

It may be avoided by using an analog low-pass filter to eliminate frequencies above $f_s/2$.

[4 points off for increasing the sampling frequency. When sampling an arbitrary waveform you don't know the maximum frequency.]

1.2 Spectral leakage is caused when frequency components are not sampled for a whole number of cycles, which results in a discontinuity between the last sample and the "next sample", which is also the first sample.

It is avoided by multiplying the sampled values by a windowing function that has zero value and zero slope at the ends of the sampling window.

[3 points off for a solution that samples for a longer time- this does not eliminate the discontinuity and long-range spectral leakage will still occur]

[3 points off if the only answer is to sample for an integer number of cycles- nothing is known about the frequency components present]

2.1 Filter gain >0.99 for frequencies <78,400 Hz

2.2 Filter gain <0.01 for frequencies >177,800 Hz

2.3 $S = M\,\Delta t = M/f_s = 2^{16}/2^{18}$ Hz = 0.25 s

2.4 H_0 corresponds to 0 Hz (d.c.); H_1 corresponds to 1/S = 4 Hz

2.5 The FFT produces coefficients H_n, where n = 0 to M–1. Therefore, the coefficient with the highest index is H_{M-1} or $H_{65,535}$, which corresponds to 4 Hz.

[2 points off for H_M and 0 Hz] [3 points off for H_M and 2^{18} Hz]

2.6 The FFT coefficient that corresponds to the highest frequency is $H_{M/2}$ or $H_{32,768}$. The corresponding frequency is (M/2)/S = 131,072 Hz

2.7 For a 4,000 Hz sinewave, the primary FFT coefficients are H_{1000} and H_{M-1000}. Additional neighboring coefficients H_{999}, H_{1001}, H_{M-999}, and H_{M-1001} are non-zero (actually half the value of the primary coefficients) due to the side lobes produced by the Hann window.

[2 points off for omitting side lobes] [2 points off for omitting H_{M-999}, H_{M-1000}, and H_{M-1001}]

2.8 For a 4,000 Hz symmetric square wave, a sequence of harmonics will appear at odd multiples of the 4,000 Hz fundamental. So H_{k1000} and $H_{M-k1000}$ would be non-zero, and the Hann side lobes would be at $H_{k1000-1}$, $H_{k1000+1}$, $H_{M-k1000-1}$, and $H_{M-k1000+1}$.

[1 point off for omitting side lobes] [3 points off for omitting harmonics]

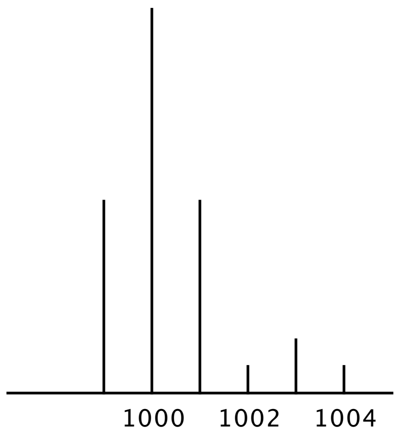
2.9 For a 4,002 Hz sinewave, H_{1000}, H_{1001}, H_{M-1000}, and H_{M-1001} would be non-zero and of equal magnitude, and the Hann side lobes would appear at H_{999}, H_{1002}, H_{M-999} and H_{M-1001}.

[1 point off for omitting side lobes] [2 points off for omitting H_{M-1000} and H_{M-1001}]

[4 points off for stating that all coefficients are non-zero]

2.10 The primary 4,000 Hz sinewave would produce non-zero values at H_{999}, H_{1000}, and H_{1001}. A second smaller sinewave of slightly higher frequency 4,000 + 4m Hz would produce non-zero values at $H_{1000+m-1}$, H_{1000+m}, and $H_{1000+m+1}$ (there are also complex conjugate coefficients at H_{M-1000}, etc.). For the smaller sinewave to appear as a separate peak, there must be a valley between the coefficient H_{1001} and the coefficient at H_{1000+m}, which requires 1000 + m > 1002, or m >2. The smallest value of m we can have is 3, which corresponds to a frequency 12 Hz above 4,000 Hz.

[4 points off for 4 Hz] [3 points off for 8 Hz] [both 12 Hz and 16 Hz were accepted]



2.11 A sinewave of frequency 4M – 84,000 Hz = 178,144 Hz will produce non-zero coefficients at H_{20999}, H_{21000}, H_{21001}, $H_{M-20999}$, $H_{M-21000}$, and $H_{M-21001}$.

$M = 2^{16} = 65{,}536$. $M - 21{,}000 = 44{,}536$.

A sinewave of frequency 84,000 Hz will produce non-zero coefficients at exactly the same frequency indexes. This is an example of how a higher frequency can alias to a lower frequency. However, the 84,000 Hz sinewave will be only slightly reduced by the anti-aliasing filter (gain >0.90, while the 178,144 Hz sinewave will be greatly reduced (gain ≈0.01). So the coefficients will be about 100 times smaller for the 178,144 Hz sinewave.

[3 points off for not stating the non-zero coefficients]

[1 point off for omitting H_{20999}, H_{21001}, $H_{M-20999}$, and $H_{M-21001}$]

[3 points off for stating that the magnitudes are the same for sampling 178,144 and 84,000 Hz]

2.12 To reduce the answer to **2.10** by a factor of two (i.e. to 6 Hz), sample for twice as long.

[2 points off for doubling the sampling frequency, which increases the number of Fourier coefficients but not the frequency spacing $\Delta f = 1/S$]

3.1 The Integral Fourier Transform will be zero except for integer multiples of the 10 kHz repeat frequency. The relative Fourier Amplitudes will depend on the waveform.

The lowest (1st) harmonic will appear at 10 kHz

The highest (1000th) harmonic will appear at 10 MHz

3.2 M = 2000 samples at f_S = 20 MHz correspond to a sampling window $S = M/f_S$ = 100μs, which is one cycle of the waveform.

The Discrete Fourier Transform will have harmonics k = 1 to 1000 at H_1 to H_{1000} and their complex conjugates at H_{1001} to H_{1999}

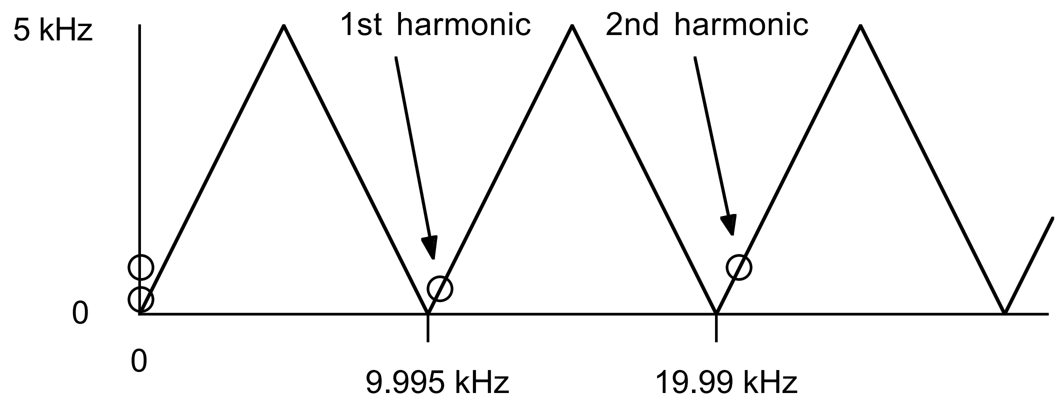
The first harmonic is H_1 at 10 kHz, which is one cycle per 100 μs

The highest harmonic H_{1000} at 10 MHz

3.3 Viewing this problem in the time domain the waveform has a period of 1/10kHz = 100μs. The sampling period of 1/9.995kHz = 100.05μs. Each sample will be progressively later by

0.05 μs. The ith sample will be at time i x 0.05μs. 2000 samples will sample one cycle of the waveform, and will be essentially the same as the values sampled in part **3.2**

Viewing this problem in the frequency domain, the first harmonic at 10 kHz will be 0.005 kHz faster than 9.995 kHz (twice the sampling frequency) and alias to 0.005 kHz.
The kth harmonic at k x 10 kHz will be k x 0.005 kHz faster than k x f_S.
The highest (1000th) harmonic at 10 MHz will 5 kHz faster than 1000 x f_S and will alias to 5 kHz.



Either approach was accepted for full credit.

Note: A sampling frequency of 9.995 kHz was chosen for this problem to avoid frequency overlap as described on page 397 of the textbook and on class handout slide 256.

EECS145M Midterm #2 class statistics:

Problem	max	average	rms
1	20	18.5	3.2
2	50	42.2	8.2
3	30	20.7	8.6
total	100	81.4	16.8

Grade distribution:

Range	number	*approximate* letter grade
30-39	1	F
40-49	0	F
50-59	0	D
60-69	2	C
70-79	4	B
80-89	3	A–, B+
90-99	5	A
100	2	A+