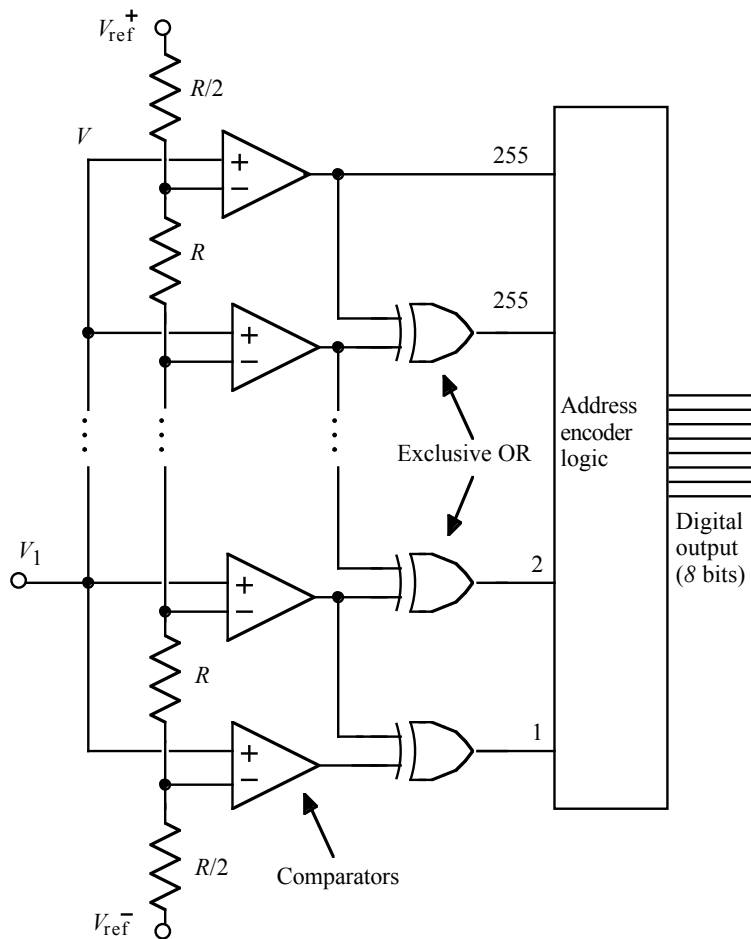


**1.1 Handshaking steps:**

When the receiver is ready, it sets “ready for data” (RD) TRUE  
 The sender detects RD TRUE and asserts the data  
 After the data have settled, the sender sets “data available” (DA) TRUE  
 The receiver detects DA true, reads the data, and sets RD FALSE  
 The sender detects RD false, and sets DA FALSE  
 [3 points off for asserting DA in a step before the step that asserts data]  
 [5 points off if the sender never asserts data]

**2.1 Flash 8-bit A/D converter:**



[4 points off for omitting the resistor chain that provides the 255 uniformly spaced reference voltages]

[4 points off for omitting the 255 comparators]

[8 points off for showing only 8 resistors, 8 comparators, and 8 output lines]

[12 points off for showing only 255 comparators and 255 digital output lines]

[4 points off for an output of 255 digital lines]

**2.2** The analog input is sent to the positive inputs of 255 comparators. A 255-resistor voltage divider provides an evenly spaced series of voltages for the negative input to the comparators. For a given analog input, the comparator output is one for all the divider voltages below the input and zero for all the voltages above. A series of exclusive or circuits identifies the highest comparator with an output of one and an encoder circuit determines the numerical value.

**3** For  $f_1 < 10$  kHz, the filter gain  $G_1 > 0.99$

Let  $f_2$  be the frequency that can alias below  $f_1$  with gain  $G_2 < 0.001$

$$f_s = 60 \text{ kHz} > f_1 + f_2$$

$$f_2 < f_s - f_1 = 50 \text{ kHz}$$

$$\text{Approximating } f_c \sim f_1, f_2/f_c \sim f_2/f_1 < 5$$

Searching for the  $n$  value where  $f/f_c < 5$  at  $G = 0.001$ , we find  $n = 6$

To check

$$\text{At } G_1 = 0.99, f_1/f_c = 0.723 \text{ so } f_c = 13.8 \text{ kHz}$$

$$\text{At } G_2 = 0.001, f_2/f_c = 3.162, \text{ so } f_2 = 43.7 \text{ kHz}$$

$f_1 + f_2 = 53.7$  kHz which is less than  $f_s$ , so the sampling frequency of 60 kHz is adequate.

Solution is  $n = 6, f_c = 13.8$  kHz

[3 points off for  $n = 4$ ] [3 points off for  $n = 10$ ]

[10 points off for a partial analysis that does not check if the gain requirements are met]

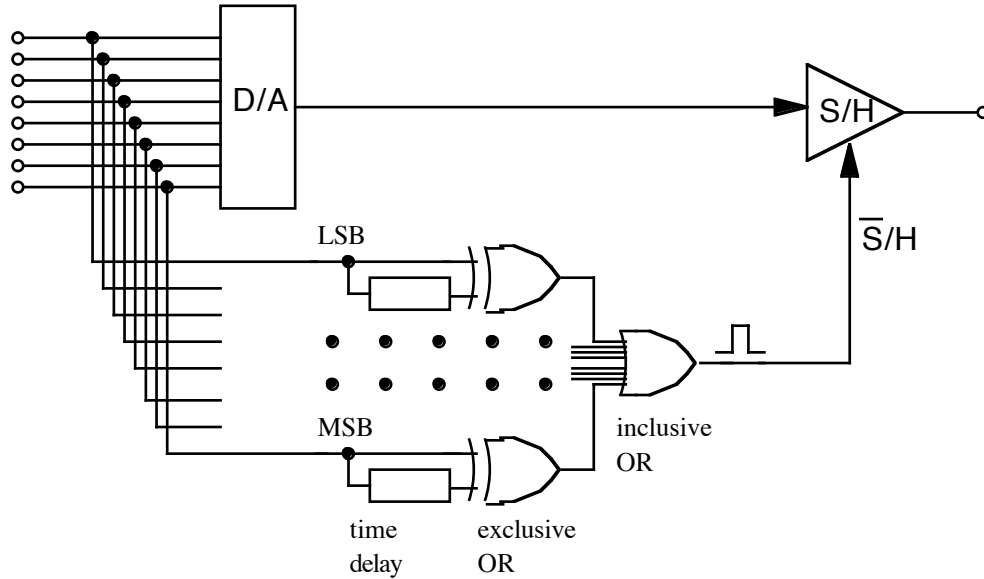
#### 4.1 D/A glitches:

When the input number changes in more than one bit, it is impossible for both bit switches to change exactly at the same time. During the brief time between the first switch change and the last switch change, an erroneous voltage will be produced.

[4 points off for settling time and no mention of bit changes]

[2 points off for not stating that the bits cannot change simultaneously]

#### 4.2



4.3 When any input bit changes, the time delay will cause the exclusive OR to change state. The pulse produced has a width equal to the time delay. The output of the inclusive OR will be high whenever any bit changes. This puts S/H into hold mode during the glitch and the glitch will not appear at the output.

[12 points off for connecting the analog output of the D/A to digital circuits]

[5 points off for including a D/A, a circuit that detects changes in the digital D/A input without specifics, and uses the signal to control a S/H]

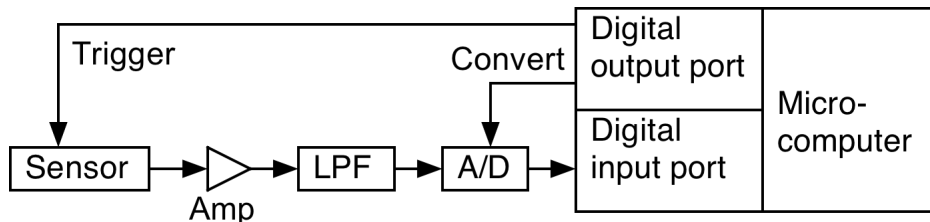
[12 points off for a circuit that has no D/A and no analog signals- all multi-bit digital signals are briefly erroneous when more than one bit changes]

[12 points off for connecting a D/A to an analog input port, not using a S/H, and not producing a deglitched analog signal]

[12 points off for an attempt that did not show a S/H after the D/A]

5 This problem was intended to find out how well you could use the anti-aliasing and Fourier techniques from this course to sample a noisy periodic signal and recover the signal with the maximum signal-to-noise ratio.

5.1



Essential components for a complete solution were

- Sensor

- Amplifier
  - Low-pass filter
  - A/D converter (S/H not required for full credit) or analog input port
  - Digital input port (if no analog input port)
  - Digital output port
  - Trigger line between digital output port and sensor (1 Hz)
  - Start conversion line between digital output port and A/D converter (~5 kHz)  
(not required if analog input port shown)
  - Analog lines connecting sensor, amplifier, filter, A/D converter or analog input port
  - Digital lines between A/D converter and digital input port  
(not required if analog input port shown)
- [2 points off for each element missing]

**5.2** The first consideration is that most of the amplifier noise was outside of the 1 Hz to 1 kHz frequency range of interest. So a low pass filter that had a gain  $G_1 = 0.99$  at  $f_1 = 1$  kHz and a gain  $G_2 = 0.0001$  at  $f_2 = 4$  kHz would effectively remove the noise from 4 kHz to 1,000 kHz. A low pass filter with  $n = 8$  and  $f_c = 1.27$  kHz would satisfy these requirements.

A sampling frequency  $f_s = f_1 + f_2 = 5$  kHz would avoid aliasing.

A raised cosine window is not necessary, since all the data of interest are periodic.

Since the sampling window was 100 s, a sampling frequency of 5,242.88 Hz would produce  $524,288 = 2^{19}$  samples for the FFT.

The process for sampling is

- 1) The computer pulses the trigger line at exactly 1 Hz
- 2) The computer pulses the A/D start conversion line at exactly 5,242.88 Hz
- 3) The computer waits for conversion to be complete (typically  $< 10$  microseconds) and reads the data (using the “conversion complete” would be good practice but was not necessary for full credit). Note that the sampling rate (5.2 kHz) is much slower than the A/D converter or the digital input port (typically  $> 100$  kHz)
- 4) After 100 seconds, the computer stops sampling and takes the FFT of the 524,288 samples.

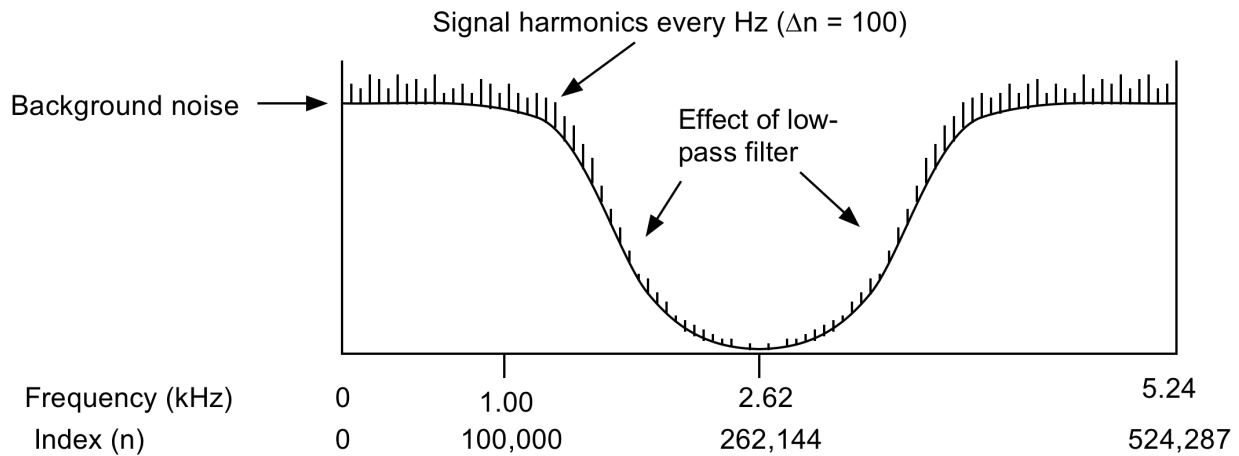
[3 points off for not specifying low pass filter design]

[3 points off for not specifying sampling frequency for triggering the analog input]

[3 points off for not triggering the sensor at 1 Hz for 100 seconds]

[3 points of for not calculating the FFT]

**5.3** The FFT will have Fourier amplitudes  $H_n$  where  $n = 0$  to 524,287. The frequency index  $n$  corresponds to frequency  $n/100\text{s} = n$  (0.01 Hz). The signal of interest will appear at frequency indexes  $100k$  ( $k =$  harmonic index of the signal) and the random amplifier noise will be spread over all Fourier amplitudes. The highest frequency of interest 1 kHz will appear at  $n = 100,000$  and at  $n = 424,287$  (complex conjugate of amplitude at  $n = 100,000$ ). The low pass filter will reduce the amplitude at higher frequencies, dropping to a gain below 0.01 at the center point  $n = 262,144$  (2.62 kHz) where  $f/f_c = 2.06$ . Frequencies above 4.24 kHz that could alias below 1 kHz are reduced by the filter to below 0.0001.



[4 points off for not showing range of FFT output in Hz or frequency index]

[4 points off for not indicating harmonics of periodic signal on a noise background]

[4 points off for not showing effect of low-pass filter]

- 5.4** The low pass filter removes most of the noise above 2 kHz but Fourier techniques can remove almost all the noise in the 1 Hz to 1 kHz frequency band of interest.

Since only the Fourier amplitudes  $H_{100k}$  contain signal information ( $k = \text{integer harmonic number}$ ) and all others contain pure noise, the latter can be set to zero.

Each Fourier harmonic  $H_{100k}$  will also contain 1% of the noise which can be estimated by averaging the other Fourier amplitudes that contain only noise.

Subtract the average noise from the harmonics  $H_{100k}$  and set all others to zero. This reduces the noise in the Fourier domain by a factor of 100.

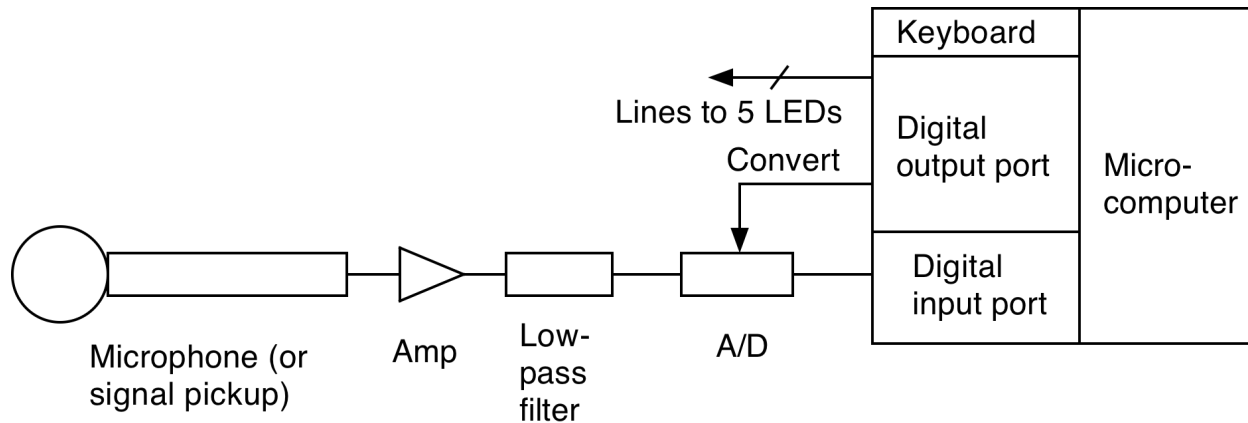
Take the inverse FFT which will contain 100 periodic identical replicas of the signal. Any one of them can be taken as the recovered signal.

[5 points off for time averaging the signal + noise before taking the FFT; this approach only reduces the noise by the square root of the number of signal repetitions ]

[5 points off for increasing the sampling window; this does not change the noise content in each Fourier amplitude but each harmonic will increase; but this is not as effective as setting the non-harmonics to zero and taking the inverse FFT]

[10 points off for not averaging in the time domain and not zeroing the non-harmonic Fourier amplitudes]

## 6.1



[3 points off for omitting the conversion input line if using an external A/D]

[3 points off for omitting LEDs]

## 6.2 Process steps

- 1) User starts the program
- 2) The program prompts the user for  $f$  and  $\Delta f$
- 3) The program asks the user to play the note
- 4) The low pass filter reduces amplifier noise and aliasing
- 4) The program continuously samples the filtered signal for a time window  $S$  to acquire  $M$  waveform amplitudes  $h_k$ ,  $k = 0$  to  $M-1$ .
- 5) The sampled values are windowed to reduce spectral leakage (in general there will not be an whole number of cycles in time window  $S$ )
- 6) Apply 5 different digital filters to the windowed data
- 7) The output of each digital filter is compared to a threshold. If the threshold is exceeded, the corresponding digital line connected to an LED is asserted. Windowing in step 5 makes it easier to identify the LED to be lit.

[3 points off for not prompting for  $f$ ,  $\Delta f$ ]

[3 points off for omitting the A/D or analog input port sampling trigger]

[a window would reduce spectral leakage but was not required since the first harmonic would still have a clear peak]

[3 points off for doing entire FFT (about 20,000 frequencies) rather than 5 digital filters]

## 6.3

The Fourier amplitude  $H_n$  at frequency  $f - \Delta f$  has index  $n = fS - 100$ .

The digital filtering that computes the real part of this amplitude is

$$\text{Re}(H_n) = \sum_{k=0}^{M-1} h_k \cos(2\pi nk / M)$$

The digital filtering that computes the imaginary part of this amplitude is

$$\text{Im}(H_n) = \sum_{k=0}^M -h_k \sin(2\pi nk / M)$$

An even more efficient approach is to use the infinite impulse response filter shown on page 418 of the textbook (lecture slide)

**145M Final Exam Grades:**

Problem	1	2	3	4	5	6	Total
Average rms							
Maximum	15	40	45	45	30	25	200

**145M Numerical Grades:**

	Short labs	Long labs	Lab Partic.	Midterm #1	Midterm #2	Final	<b>Total</b>
Average rms			100				
Maximum	100	400	100	100	100	200	<b>1000</b>

**145M Letter Grade Distribution**

Letter Grade	Course Totals (1000 max)
A+	
A	
A-	
B+	
B	
B-	
C+	
C	
C-	
D+	
D	
D-	

\* Graduate student