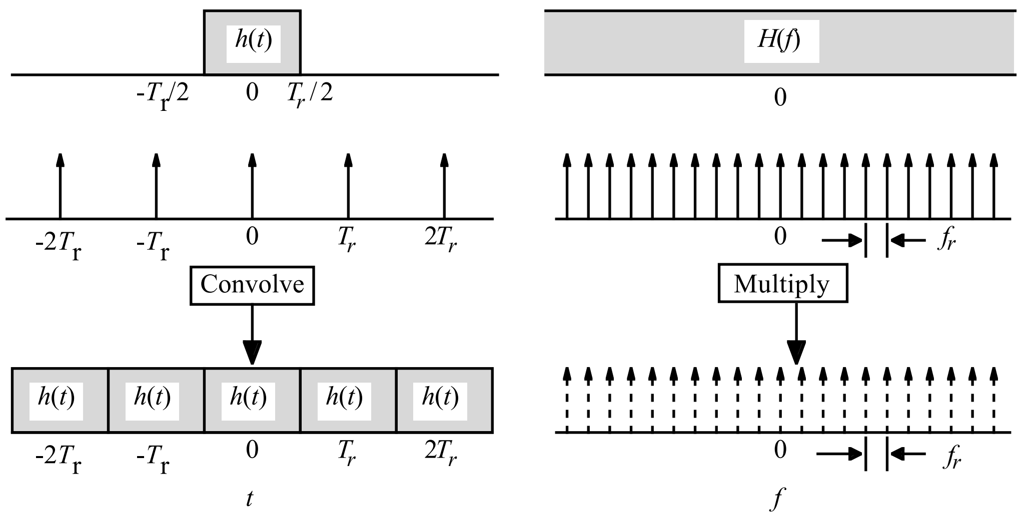
Solutions for Midterm #2 - EECS 145M Spring 2009

1.1 Fourier Convolution Theorem: The Fourier transform of the convolution of two functions is the product of their Fourier transforms (Textbook p 391, lecture slide #218).

[Note: no deduction for switching the answers for 1.1 and 1.4. It is more important to know the two theorems than to know exactly what they are called.]

[6 points off for stating the same theorem for 1.1 and 1.4]

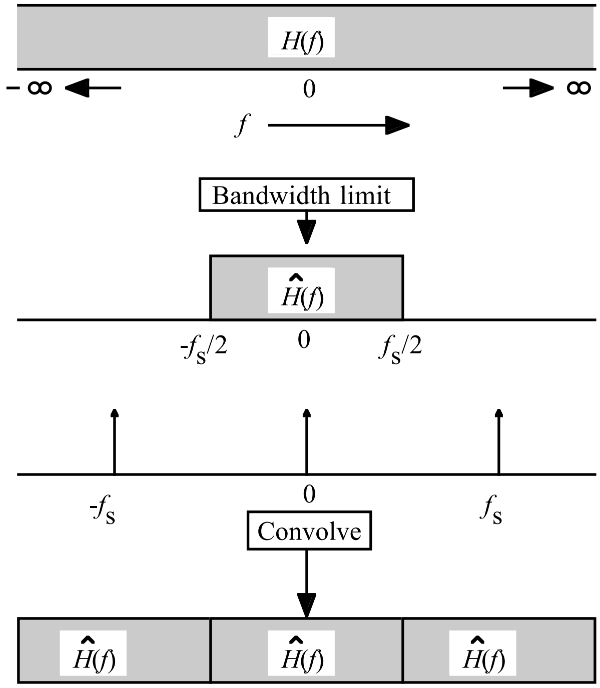
1.2 A periodic waveform is the convolution of a waveform with an infinite train of delta functions. The Fourier transform of the latter is an infinite train of delta functions in frequency. By the Fourier convolution theorem, the Fourier transform of a periodic waveform is the product of its Fourier transform with an infinite train of delta functions in frequency. The resulting Fourier transform has non-zero values only at discrete frequencies (Textbook Figure 5.26, lecture slide #224).



1.3 Nyquist sampling theorem: To recover a waveform from its sampled values, the sampling frequency must be at least twice the highest frequency in the signal (Textbook p 404, lecture slide #177).

1.4 Fourier Frequency Convolution Theorem: The Fourier transform of the product of two functions is the convolution of their Fourier transforms (Textbook p 394, lecture slide #225)

1.5 Periodically sampling a waveform is equivalent to multiplying the waveform by a continuous train of delta functions. The Fourier transform of the latter is a continuous train of delta functions with a spacing equal to the sampling frequency. By the Fourier frequency convolution theorem, the Fourier transform of the product is the convolution of the two Fourier transforms. The result is an infinite summation of copies of the Fourier transform of the waveform, each copy shifted by integer multiples of the sampling frequency. Any frequencies outside $-f_s/2$ and $+f_s/2$ will overlap other frequencies, resulting in frequency aliasing (Textbook p 396, lecture slide #230).



[2 points off for not stating that sampling means multiplying by a train of delta functions in the time domain]

[2 points off for not stating that the Fourier transform is convolved with a train of delta functions in the frequency domain.

[2 points off if overlap in frequency not stated as the cause of aliasing]

[4 points off for restating the Nyquist theorem]

2.1 For n = 8 and G = 0.99, f/fc = 0.784. Since fc = 20 kHz, gain is greater than 0.99 for frequencies below f = 15.68 kHz

2.2 For n = 8 and G = 0.001, f/fc = 2.371. Since fc = 20 kHz, gain is less than 0.001 for frequencies above f = 47.42 kHz

2.3 20 kHz will appear at $H_{20,000}$ with side lobes at $H_{19,999}$ and $H_{20,001}$ due to the Hann window.

It will also appear at $H_{45,536}$, $H_{45,536}$, and $H_{45,536.}$ (frequency index 65,536 – 20,000 = 45,536 plus Hann side lobes).

The amplitudes are proportional to the anti-aliasing filter gain which is 0.707 at 20 kHz.

40 kHz will appear at $H_{40,000}$ with side lobes at $H_{39,999}$ and $H_{40,001}$ due to the Hann window.

It will also appear at $H_{25,536}$, $H_{25,536}$, and $H_{25,536.}$ (frequency index 65,536 – 40,000 = 25,536 plus Hann side lobes).

The amplitudes are proportional to the anti-aliasing filter gain which is 0.00390 at 40 kHz.

[3 points off if no 40 kHz]

[2 points off if no Hann side lobes]

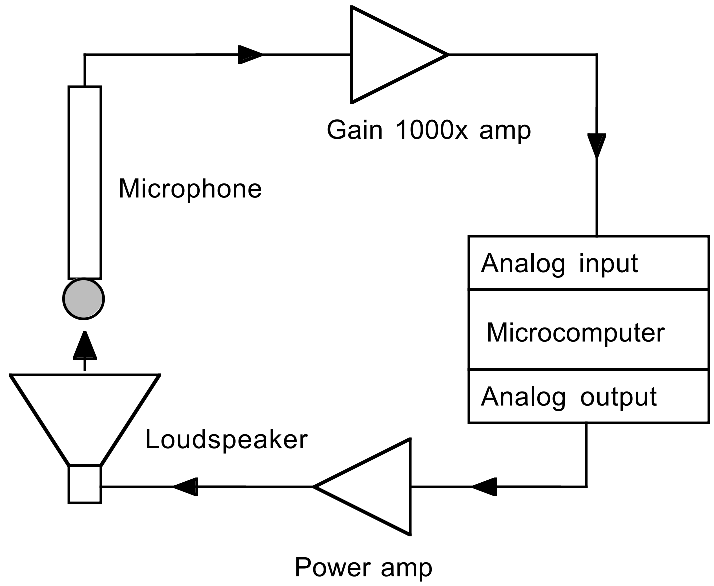
[2 points off if no relative amplitudes]

2.4 If the Hann window were not used, there would be no side lobes.

The 20 kHz would produce Fourier components at $H_{20,000}$ and $H_{45,536}$ with relative amplitudes of 0.707

The 20 kHz would produce Fourier components at $H_{40,000}$ and $H_{25,536}$ with relative amplitudes of 0.00390

3.1



[Note: some higher frequencies are introduced because of the step changes at the analog output frequency, but an anti-aliasing filter was not required for full credit]

[2 points off if no microphone amplifier (5 mV << 5 V)]

[2 points off if no loudspeaker power amplifier (analog output ports cannot deliver much current)]

3.2 H_1 corresponds to 4 Hz; H_{1000} corresponds to 4 kHz

3.3

1) Calculate the FFT of the pseudo-random waveform (H_{in})
2) Send the pseudo-random waveform to the loudspeaker and simultaneously sample the microphone signal (H_{out})
3) The frequency response of the loudspeaker is $FFT(H_{out})/FFT(H_{in})$
4) Since the compensating digital filter needs to have a frequency response that is the inverse of this, the digital filter is computed as $FFT^{-1}[FFT(H_{in})/FFT(H_{out})]$

   [2 points off for omitting step 4]

3.4 The compensating digital filter could be checked by using it on any waveform that has a broad frequency content and see if sending it to the loudspeaker results in a microphone signal that is close to the initial waveform

3.5 Send the 16,384 pseudo-random waveform to the loudspeaker 100 times, simultaneously sample the microphone signal, and take the FFT of the 1,638,400 samples. The signal will appear only in the harmonics, which occur only at frequency indices 100, 200, etc. All others are pure noise and can be ignored. Taking every 100th results in a Fourier transform with 16,384 elements containing 1% of the noise of a single repetition of the pseudo-random sequence.

This is better than averaging 100 noisy waveforms, which reduces the noise by only a factor of ten.

[2 points off for averaging the waveforms rather than setting the non-harmonic Fourier amplitudes to zero]

EECS145M Midterm #2 class statistics:

Problem	max	average	rms
1	30	22.1	3.6
2	25	21.1	3.2
3	45	34.0	7.3
total	100	77.1	10.5

Grade distribution:

Range	number	*approximate* letter grade
40-49	0	F
50-59	1	D
60-69	2	C
70-79	3	B
80-89	7	A–, B+
90-99	1	A
100	0	A+