`Solutions for Midterm #2 - EECS 145M Spring 2008

1.1 Frequency aliasing is caused by sampling at a frequency f_s that is less than twice the highest frequency f_{max} in the waveform.

It may be avoided by using an analog low-pass filter to eliminate frequencies above $f_s/2$.

[4 points off for increasing the sampling frequency, since f_{max} is not known]

1.2 Spectral leakage is caused when frequency components are not sampled for a whole number of cycles, which results in a discontinuity between the last sample and the “next sample”, which is also the first sample.

It is avoided by multiplying the sampled values by a windowing function that has zero value and zero slope at the ends of the sampling window.

[3 points off for a solution that samples for a longer time- this does not eliminate the discontinuity and long-range spectral leakage will still occur]

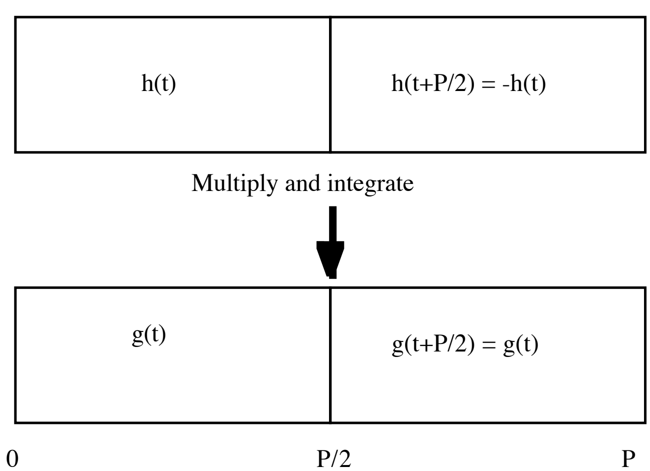
[3 points off if the only answer is to sample for an integer number of cycles- nothing is known about the frequency components present]

2.1 H(f) can only be non-zero at multiples of the repeat frequency = n/P

[4 points off for describing or sketching only the first harmonic]

2.2 All the components in the Fourier integral have periods that are integer multiples of P, so integrating from 0 to P is all that is necessary.

$$H(f) = \int h(t)g(t)dt \quad g(t) = \exp(-j2\pi nt / P)$$



$g(t) = \exp(-j2\pi nt/P)$

For n even, $g(t+P/2) = \exp(-j2\pi n[t+P/2]/P) = \exp(-j2\pi nt/P - j2\pi n/2) = \exp(-j2\pi nt/P) = g(t)$

For even n, the harmonic factor $\exp(-j2\pi nt/P)$ is the same in the interval from $t = 0$ to $P/2$ as it is in the interval from $t = P/2$ to P. However, $h(t)$ has the opposite sign from $t =0$ to $P/2$ as it does from $t = P/2$ to P and the two inner products cancel.

[Full credit for showing that the integral from 0 to P could be split into two integrals that cancel][4 exams out of 17 had part **2.2** correct]

[5 points off for a valiant but incomplete attempt]

[the most common approach was to integrate both $h(t)g(t)$ and $h(t+P/2)g(t+P/2)$ from 0 to P, without following up with a convincing argument that each was zero for even n.]

3.1 Need to show Butterworth filter, S/H amplifier, A/D, timer, computer

[2 points off for each missing component]

$G_1 = 0.99\ f_1/f_c = 0.823$, $f_c = 10$ kHz/0.823 = 12.15 kHz

3.2 $G_2 = 0.001\ f_2/f_c = 1.995$; $f_2 = 24.24$ kHz

$f_s > f_1 + f_2 = 34.24$ kHz

[3 points off for $f_s > 2f_c = 24$ kHz. This is not sufficient]

[5 points off for fs > $2f_1 = 20$ kHz. This assumes a perfect rejection above 10 kHz.

3.3 $T = 1/(f * 2^{17} * \pi) = 1/(10 \text{ kHz} * 131{,}072 * 3.1416) = 0.24$ ns

[2 points off for a minor numerical error]

[5 points off for not solving for T]

3.4 For S = 0.4 s, $\Delta f = 2.5$ Hz.

[S = 0.3 to 0.5 seconds was accepted for full credit]

[5 points off for S = 0.1 seconds]

[3 points off for S = 0.2 seconds]

3.5 34.24 kHz x 0.4 sec = 13,696 (next power of 2 is 16,384).

3.6

Set counter to sample A/D for 0.4 sec at 34.24 kHz

Start sampling output of Butterworth filter

When done, multiply samples by Hann window

Take FFT

[2 points off for no Hann or similar window]

[2 points off for not giving the frequency that corresponds to H_n]

Each Fourier frequency component H_n corresponds to 2.5xn Hz.

EECS145M Midterm #2 class statistics:

Problem	max	average	rms
1	20	19.5	1.4
2	20	13.5	3.2
3	60	51.5	6.4
total	100	84.6	8.3

Grade distribution:

Range	number	*approximate* letter grade
65-69	1	C+
70-74	1	B-
75-79	3	B
80-84	1	B+
85-89	8	A-
90-94	1	A
95-99	2	A+
100	0	A+