PROBLEM 1 (20 points)

1.1 (10 points) When periodically sampling an arbitrary waveform, what causes frequency aliasing and how can it be reduced?

1.2 (10 points) When periodically sampling an arbitrary waveform and computing its Fourier transform, what causes spectral leakage and how can its long-range effects be reduced?
**PROBLEM 2** (20 points)

2.1 (10 points) If a waveform $h(t)$ is periodic with period $P$, what does its Fourier transform $H(f)$ look like? (Describe or sketch below)

2.2 (10 points) Use the definition of the Integral Fourier Transform (see equation sheet) to prove that if a periodic waveform of period $P$ is half-period symmetric $h(t) = -h(t+P/2)$, then the Fourier components $H(f)$ are zero for even harmonic frequencies $f_n = n/P$ ($n$ even). *Hint:* Integrate from $t = 0$ to $t = P$. 

PROBLEM 3 (60 points)
Design a Fast Fourier Transform frequency analysis system that meets the following requirements:

- The frequency range of interest is 0 to 10 kHz
- The system must be able to clearly distinguish two harmonic components that differ in frequency by 10 Hz.
- Any frequency components that could alias to 10 kHz or below are reduced to 0.1% by a low-pass Butterworth filter
- The Butterworth filter gain is > 0.99 at 10 kHz and below

Assume that you will use the following:

- A low-pass Butterworth filter of order 10
- A 16-bit successive approximation A/D converter
- A timer that controls the A/D converter at a rate and of your choosing and for a number of samples of your choosing
- A raised cosine window
- Any other components from C145M that are necessary

3.1 (10 points) Sketch a block diagram of your system (Include all major components and specify the corner frequency of the Butterworth filter).
3.2 (10 points) What is the minimum sampling frequency necessary to satisfy the requirements? *Hint:* Don’t forget to consider the effect of the Butterworth filter.

3.3 (10 points) What is the analog input time jitter that corresponds to a jitter of 1/2 least significant bit at the output of the A/D converter for a full-scale harmonic at 10 kHz?

3.4 (10 points) What is the minimum duration of the sampling necessary to meet the requirements? *Hint:* Consider the effect of the Hann window.
3.5  (10 points) What is the minimum number of sampled values necessary to meet the requirements?

3.6  (10 points) List the steps (hardware and software) necessary to perform this task. (You can use the values from your answers to 3.2, 3.4, and 3.5). To what frequency does the Fourier coefficient $H_n$ correspond?