Solutions for Midterm #2 - EECS 145M Spring 2005

Problem 1

Since 5 symmetric square waves were sampled, we expect non-zero magnitudes at frequency indexes at odd multiples of 5 and 128 minus odd multiples of 5. These are n = 5 and 123 (first harmonic at 15 Hz), n = 15 and 113 and (third harmonic at 45 Hz), n =25 and 103 (fifth harmonic at 75 Hz), etc.

The additional large magnitudes at n = 20 and 108 correspond to a pure 60 Hz harmonic, which must be due to power line interference in the lab.

If magnitude at n = 20 were due to distortion generating even harmonics, additional magnitudes at n = 10, 30, 40, 50, etc. would also be expected.

[2 points off if odd harmonics not identified]

[3 points off if magnitudes above index 64 not explained]

[4 points off if 60 Hz electromagnetic interference not identified]

2 As an initial approximation to the filter order n, find the first entry down the G = 0.001 column for which $f/f_c < (f_s - f_1)/f_1$. The result is $n = 6$.

For $n = 6$ and $G_1 = 0.90$, the table shows $f_1/f_c = .886$, so $f_c = 22.6$ kHz.

For $G_2 = 0.001$, the table shows $f_2/f_c = 3.162$, so $f_2 = 71.3$ kHz.

$f_1 + f_2 = 91.3$ kHz, so the 100 kHz sampling frequency is adequate.

The solution is $n = 6$ and $f_c = 22.6$ kHz.

[3 points off for n = 4] [3 points off for n = 10]

3a Multiplying the data with a square window convolves the true frequency spectrum with the Fourier transform of the square wave, which produces long-range spectral leakage that falls off as 1/freqency.

[4 points off for explaining what is happening in the time domain but not in the frequency domain]

[4 points off for a qualitative description of spectral leakage]

[6 points for describing the FFT of the true frequency spectrum but not the effects of the rectangular sampling window]

3b To recover the original square-wave-windowed time samples, just use the inverse transform. Then multiply the time samples with the raised cosine window and take the fft.

A less efficient solution (N^2 steps rather then N logN steps) was to convolve the original FFT with the FFT of the raised cosine

[5 points off for ***multiplying*** the original FFT by the FFT of the raised cosine]

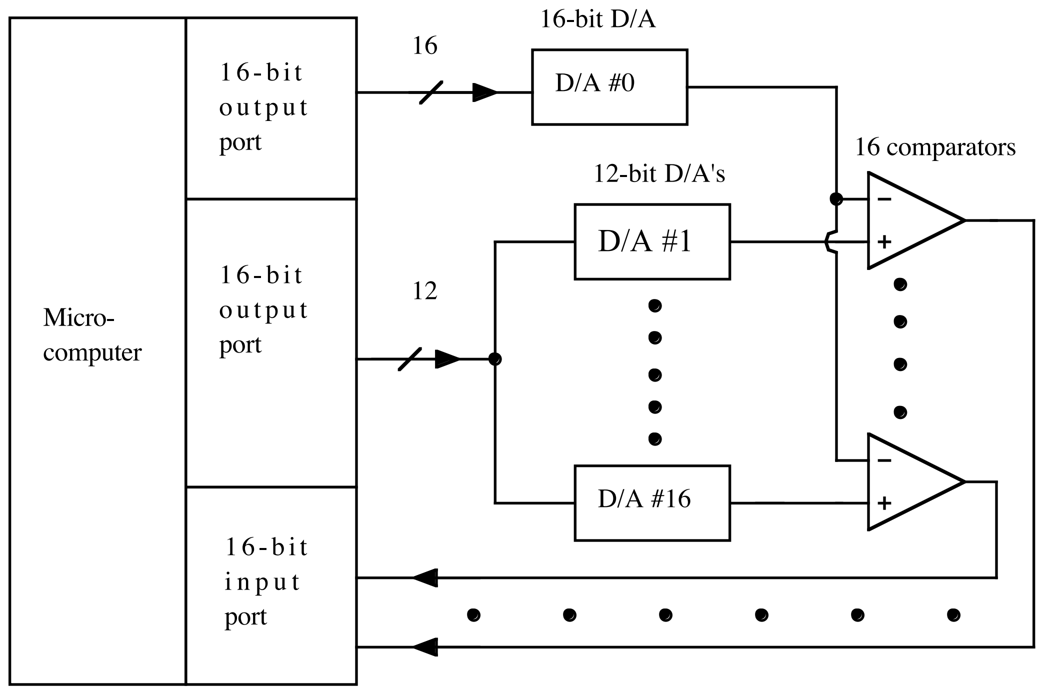
[10 points off for only fixing up selected "features" of the original FFT]

3c Multiplying the data with the raised cosine window convolves the true frequency spectrum with the Fourier transform of the raised cosine window, which produces short-range spectral leakage (nearest-neighbor frequency amplitudes) but very small long-range spectral leakage.

[5 points off for not explaining what is happening in the frequency domain]

[7 points for describing the FFT of the true frequency spectrum but not the effects of the raised cosine window]

4a



[5 points off if the 16-bit reference D/A and the 12-bit D/As do not have independent inputs]

4b

<u>The solutions in words:</u>

The program sends all possible (4096) 12-bit integers I1 to all (16) 12-bit D/A converters and for each value I1 compares the outputs to the output of a 16-bit D/A converter. To cycle both the 16-bit D/A and the 12-bit D/A inputs, two nested loops are required. The comparators are used to determine what 16-bit D/A input value gives an output that is close to each of the 4096 x 16 outputs and that value is stored in a 4096 x 16 array.

For each 12-bit D/A:

The maximum absolute error is the maximum difference between the ideal value I1*16 and the corresponding value in the stored array

The maximum linearity error is the maximum difference between the stored values and a line between the end points I1 = 0 and I1 = 4095

The maximum differential linearity error is the maximum difference between the stored values for I1 and I1+1 and the average difference.

<u>The solution as a program:</u>

1 Set I1=1
2 output I1 to all 12-bit D/A converters
3 Set I2 =1
4 Output I2 on the 16-bit D/A converter and wait for them to settle
5 Input the 16 comparator outputs. When I2 is much lower than 16*I1, all the comparator outputs will be 1. If the bit for comparator I3 has just changed to 0 set Data(I3,I1) = I2
6 Set I2 = I2 +1 and loop back to step 5 until all comparators = 1 or until I2 = 2^{16} =65,536
7 Set I1 = I1 +1 and loop back to step 2 until I1 = 2^{12} = 4096

8 The ideal value of Data(I3,I1) is I1*16 so the maximum absolute error of D/A #I3 is the maximum value of Data(I3,I1) – I1*16

9 The maximum linearity error of D/A #I3 is the maximum difference between Data(I3,I1) and a straight line between Data(I3,0) and Data(I3,4095)

10 The maximum differential linearity error of D/A #I3 is the maximum difference between Data(I3,I1) – Data(I3,I1+1) and the average step size

[5 points off if the 16-bit D/A and the 12-bit D/As inputs are not cycled in two nested loops]

[5 points of if not steps are given for the calculation of maximum absolute error, maximum linearity error, and maximum differential linearity error]

[Many incorrect answers assumed that the comparators gave a quantitative digital output, when all they do is give a single bit output that says which analog input is larger]

[Some answers speculated that the comparator could determine if two analog inputs were equal. While two digital inputs can be exactly equal, two static analog inputs can never be exactly equal]

4c The quantities in 4b can be determined to an accuracy of ±1/16 LSB.

[2 points off for ±1/2 or ±1 LSB since the question referred to the accuracy of the 12-bit D/As, not the 16-bit D/A]

EECS145M Midterm #2 class statistics:

Problem	max	average	rms
1	15	8.6	3.7
2	15	14.3	1.8
3	35	24.5	7.1
4	35	27.4	5.0
total	100	74.8	11.5

Problem 1 was Problem 4 of the 2002 Final Exam. Students that year averaged 11.9/15.

Problem 3 was Problem 5 of the 2002 Final Exam. Students that year averaged 29.4/40 (which is equivalent to 25.7/35).

Grade distribution:

Range	number	*approximate* letter grade
45-49		D+
50-54	1	C-
55-59	1	C
60-64	3	C+
65-69	4	B-
70-74	3	B
75-79	5	B+
80-84	3	A-
85-89	4	A
90-94	0	A+
95-99	2	A+
100	0	A+