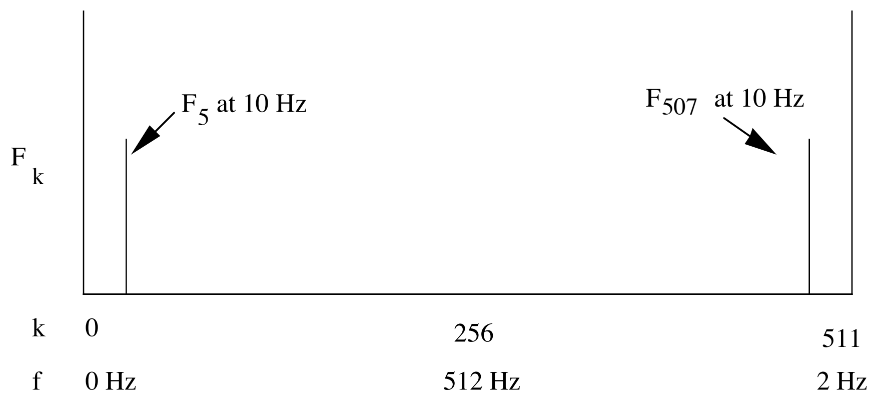
Solutions for Midterm #2 - EECS 145M Spring 2003

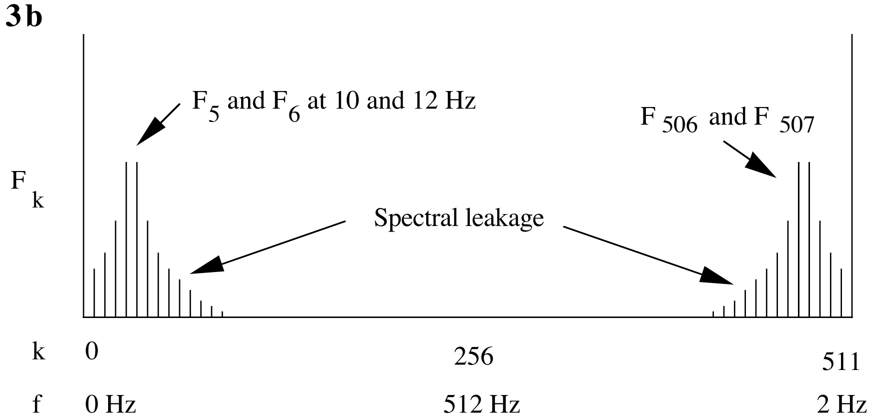
1a See text, page 416.
1b See text, page 415
2a See text, page 391
2b See text, page 394
2c See text, page 404

3a Sampling at 1024 Hz for S = 0.5 seconds produces 512 samples. The FFT produces 512 coefficients, and the magnitude F_k of the kth coefficient corresponds to a frequency of k/S = $2k$ Hz.



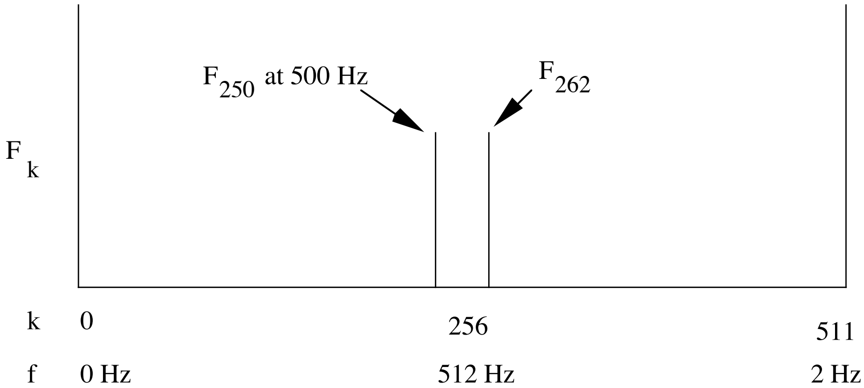
$F_0 = 0$
F_5 and $F_{512\text{-}5}$ are the only non-zero Fourier coefficient
No aliasing or spectral leakage
[4 points off if frequency and index scales not shown]
[2 points off if frequency scales in Hz not shown or 1024 Hz is at right hand edge]

3b



F_5, F_6, $F_{512\text{-}6}$, and $F_{512\text{-}5}$ are non-zero
Spectral leakage, because 5.5 cycles are sampled in 0.5 sec
Values around these have spectral leakage, falling off as $1/f$
No aliasing, since 11 Hz < $F_s/2$ = 512 Hz
[2 points off if spectral leakage not shown]

3c



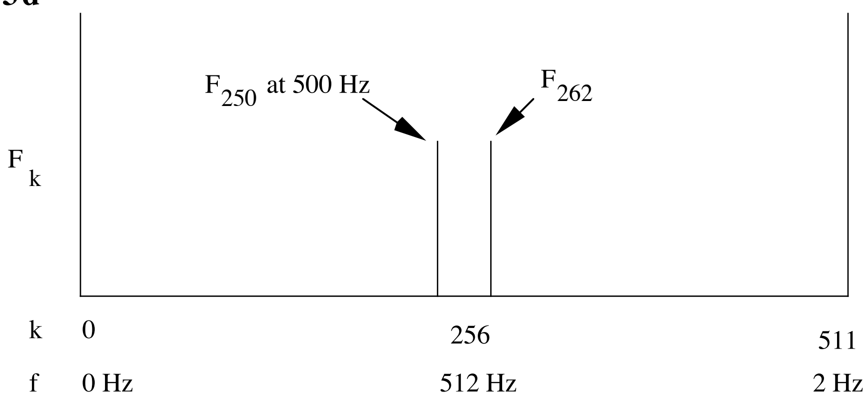
250 cycles are sampled in 0.5 sec- no spectral leakage

H_{250} and $H_{512-250}$ (H_{250} and H_{262}) are non-zero

No aliasing because 500 Hz is below the Nyquist frequency of 1024 Hz/2 = 512 Hz

No spectral leakage because exactly 250 cycles are sampled

3d



262 cycles are sampled in 0.5 sec- no spectral leakage

H_{262} and $H_{512-262}$ (H_{250} and H_{262}) are non-zero

No spectral leakage because exactly 262 cycles are sampled

Aliasing occurs because 524 Hz is above the Nyquist frequency of 1024 Hz/2 = 512 Hz

Fourier magnitudes for 524 Hz look exactly like Fourier magnitudes for 500 Hz

3e a: exactly 5 cycles of sinewave sampled- no aliasing, no spectral leakage

b: 5.5 cycles of sinewave sampled- no aliasing, but spectral leakage that can be fixed by using a raised cosine window or by increasing the sampling time to 1 sec

c: 500 Hz sinewave sampled at 1024 Hz- no spectral leakage, no aliasing

d: 524 Hz sinewave sampled at 1024 Hz- no spectral leakage, but aliasing because 524 Hz signal frequency is above 1/2 the sampling frequency. The samples of the 524 Hz sinewave look exactly like the samples of the 500 Hz sinewave. Fixed by sampling faster than 2 x 524 Hz. [1 point off if the only fix is to use an anti-aliasing filter, which would eliminate the signal]

4

1 Set width T_0 of pulses to $1/(2 f_{max})$ or less
2 Set the repetition frequency of the pulse generator to f_0
3 Connect the output of the pulse generator to the input of the analog filter
The Fourier transform of this input signal is the sinc function (given on the equation sheets)

$$H(f_n) = AT_0 \frac{\sin(\pi T_0 f_n)}{\pi T_0 f_n} \qquad f_n = n f_0$$

4 Sample the output for one repetition period ($1/f_0$) at a sampling frequency $2 f_{max}$ or above
5 Take the FFT
6 Each Fourier coefficient F_n is the amplitude of the output waveform at frequency nf_0
7 The amplitude gain of the analog filter is the output amplitude divided by the input amplitude

$$\frac{V_{out}}{V_{in}} = \frac{|F_n|}{|H(f_n)|}$$

[2 points off for not specifying the pulse width in terms of f_{max}]
[2 points off for not specifying the sampling frequency in terms of f_{max}]
[2 points off for not specifying the sampling window or pulse frequency in terms of f_0]
[3 points off if analog filter to be measured is not used in the procedure]
[3 points off if no formula is given for calculating voltage gain or if the formula is incorrect- one common error was to divide the FFT of the sampled filter output by the input voltage amplitude]

Midterm #2 class statistics:

Problem	max	average	rms
1	12	9.1	4.2
2	18	17.6	1.1
3	50	44.4	7.6
4	20	13.9	3.4
total	100	84.8	12.1

Grade distribution:

Range	number	*approximate* letter grade
40-49	1	F
50-59	0	F
60-69	0	D
70-79	3	C
80-89	6	B
90-99	6	A
100	1	A+