Solutions for Midterm #2 - EECS 145M Spring 2002

1a Aliasing is caused by sampling at a frequency that is less than twice the highest frequency present in the waveform.

It is prevented by filtering out all frequencies higher than 1/2 the sampling frequency.

1b Spectral leakage is caused when the last time samples do not join smoothly with the first samples.

If the waveform is periodic, spectral leakage is prevented by sampling a whole number of periods. Using a raised cosine window was also accepted for full credit.

[3 points off for increasing the window width S]

If the waveform is non-periodic, spectral leakage can be greatly reduced by using a raised cosine window

[3 points off for increasing the window width S or the sampling frequency fs]

[3 points off for adding leading and trailing zeros]

2a The waveform is sampled at 32,768 Hz for two 2 seconds, yielding 65,536 samples. The fft input consists of 65,536 real and 65,536 imaginary numbers, for a total of 131,072 numbers.

The output also has 131,072 numbers.

[1 point off for answering 65,536 to both]

2b H_0 corresponds to 0 Hz (dc). H_1 corresponds to 0.5 Hz (one cycle per S = 2 sec)

2c $F_{32,768}$ corresponds to the highest frequency of 16,384 Hz.

2d The pure 1000 Hz tone will appear at F_{2000} and $F_{63,536}$. All other Fourier components should be zero.

[Note: F_0 was optional. 2 points off if no complex conjugate.]

[2 points off for H_{500} and H_{M-500}]

[2 points off for H_{k2000} and $H_{M-k2000}$ since for an undistorted tone there are no higher harmonics]

2e The kth harmonic will appear in Fourier coefficients F_{k2000} and $F_{65,536-k2000}$.

[Note: 2 points off if complex conjugates missing.]

2f F_{2000} and F_{2001} are equal because the pure 1000.25 Hz tone lies exactly between them. Their complex conjugates are at $F_{63,535}$ and $F_{63,536}$. Neighboring values are small but non-zero due to spectral leakage.

[Note: 2 points off if harmonics such as F_{4000} listed. 2 points off if small values called zero. 2 points off if no complex conjugates. 2 points off if F_{2001} not listed.]

2g F_{2000} and F_{2001} (and their complex conjugates at $F_{63,535}$ and $F_{63,536}$) are about the same because the pure 1000.25 Hz tone lies exactly between them. F_{1999} and F_{2002} (and their complex conjugates at $F_{63,534}$ and $F_{63,537}$) are about half of F_{2000} and F_{2001} due to the short-range spectral leakage of the raised cosine window. All other Fourier coefficients are essentially zero.

[2 points off if short-range spectral leakage of raised cosine window not mentioned]

3a v = 3 m/s f = 100 kHz (1 + 3/300) = 101,000 Hz

v = 30 m/s f = 100 kHz (1 + 30/300) = 110,000 Hz

v = 30.3 m/s f = 100 kHz (1 + 30.3/300) = 110,100 Hz

v = 60 m/s f = 100 kHz (1 + 60/300) = 120,000 Hz

The exact calculation gives 101,010; 111,111; 111,235; and 125,000 Hz; both were OK.

3b Δf = 100 Hz, so the minimum length of the sampling window must be $S = 1/\Delta f = 0.01$ s

3c The spectral leakage from 100 kHz to 101 kHz will be severe, so a raised cosine window would be useful in nearly eliminating it. The broadening caused by the raised cosine window will extend only to nearest neighbor frequencies 100 Hz away and will not interfere with an echo at 101 kHz. So there is no need to increase S.

An alternative, a precise number of 100 kHz cycles could be sampled. The much smaller echo signal would still have spectral leakage, but that would be OK.

3d Initially trying n =6, a gain of 0.9 at 120 kHz results in f1/fc = 0.886 and the corner frequency is thus 135.4 Hz. G2 = 0.01 occurs at f2/fc = 2.154, yielding f2 = 291.7 kHz. f1 + f2 = 411.7 kHz, just above our sampling frequency of 409.6 kHz. This does not meet the requirement, so we should increase n.

Trying n =8, and a gain of 0.9 at 120 kHz results in f1/fc = .913 and the corner frequency is thus 131.9 Hz. G2 = 0.01 occurs at f2/fc = 1.778, yielding f2 = 234.5 kHz. f1 + f2 = 354.5 kHz, well below our sampling frequency of 409.6 kHz.

[3 points off for not checking that fs > f1 + f2]

3e Sampling at 409.6 kHz for 0.01 s would produce 4096 samples.

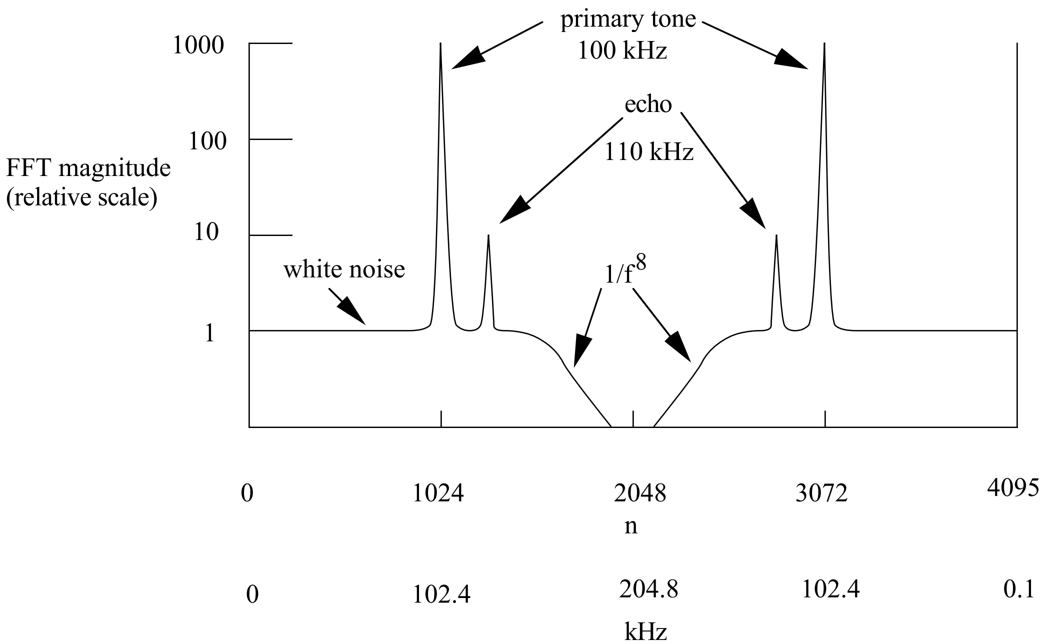
3g The echo is 100 times smaller than the 100 kHz tone and the white noise is 10 times smaller than the peak echo. The low pass filter falls off as $1/f^8$.

[2 points off if no FFT frequency index] [2 points off if no frequency scale in Hz]

[2 points off if white noise not shown] [4 points off if primary tone or echo not shown]

[3 points off for blank vertical scale] [2 points if vertical scale is not numbered]

[2 points off if effect of filter not shown]



Midterm #2 class statistics:

Problem	max	average	rms
1	16	15.1	1.3
2	34	27.4	3.9
3	50	44.2	7.6
total	100	86.7	10.1

Grade distribution:

Range	number	*approximate* letter grade
51-55	0	
56-60	0	
61-65	1	D
66-70	1	D
71-75	0	C
76-80	0	C
81-85	1	B-
86-90	4	B
91-95	5	A
96-100	0	