Solutions for Midterm #2 - EECS 145M Spring 2001

1a  Successive approximation A/D

1b
1. set all bits to zero
2. set index \( i = N \) (MSB)
3. set bit \( i \) to one
4. send bit pattern to D/A
5. if analog input is less than D/A output, set bit \( i \) to zero
6. \( i = i - 1 \)
7. return to step 3 (quit if \( i = 0 \))

2a  Flash A/D

\[
V_{\text{ref}} + (2^N - 15) \Delta V = V_{\text{ref}} - 0.5 \Delta V
\]
2b

1. Analog input is sent to the (+) inputs of $2^{N-1}$ comparators.
2. (-) inputs of comparators connected to points between resistors connected in series.
3. Comparator outputs are sent to a circuit that determines the $N$-bit address of the highest comparator whose output is one.
4. The $N$-bit address is the converted output.

3a

An infinite periodic series of square pulses of width $T_0$ and period $T_r$ is the convolution of the square wave $h(t)$ with an infinite periodic series of delta functions:

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_r)$$

By the Fourier convolution theorem, the Fourier transform of $h(t)$ convolved with $g(t)$ is the simple product of the individual Fourier transforms $H(f)$ and $G(f)$:

$$G(f)H(f) = \sum_{n=-\infty}^{\infty} \frac{\sin(\pi f T_0)}{\pi f T_0} \delta(f - nf_r) \quad f_r = 1 / T_r$$

This Fourier transform has the envelope of $H(f)$ but is non-zero only at integer multiples of the repeat frequency $f_r$.

3b For $T_0 = 1 \mu s$ and $T_r = 1 \text{ ms}$

The Fourier transform is non-zero only at integer multiples of the repeat frequency $f_r = 1 \text{ kHz}$. 
4a

\[ \mu \text{ computer} \quad \text{set } Tr \quad \mu \text{ computer} \quad \text{FFT} \]

4b  Want filter gain \( G_1 > 0.999 \) for frequencies \( f_1 < 100 \text{ kHz} \).

From equation sheet, an 8-pole filter has a gain of 0.999 at \( f/f_c = 0.678 \)

Solve for \( f_c = f_1 / 0.678 = 147.5 \text{ kHz} \)

Want filter gain \( G_2 < 0.01 \) at the lowest frequency \( f_2 \) that could alias below \( f_1 = 100 \text{ kHz} \)

From equation sheet, an 8-pole filter has a gain of 0.01 at \( f/f_c = 1.778 \)

Solve for \( f_2 = 1.778 f_c = 262 \text{ kHz} \)

\( f_2 \) aliases to \( f_1 \) when \( f_s = f_1 + f_2 \)

To avoid aliasing we want \( f_s > 100 \text{ kHz} + 262 \text{ kHz} = 362 \text{ kHz} \)

[the requirement that \( f_s > 2 f_2 = 524 \text{ kHz} \) is more conservative than necessary but was accepted with no deduction]

4c  Since we only need Fourier magnitudes at multiples of 100 Hz, the series of 1 \( \mu \text{s} \) pulses needs to contain harmonic frequencies only at multiples of 100 Hz. By choosing a pulse repetition period \( Tr = 0.01 \text{ seconds} \), the series of 1 \( \mu \text{s} \) pulses contains a fundamental frequency of 100 Hz and higher harmonic multiples of 100 Hz.

Since the number of samples \( M \) is equal to the number of Fourier magnitudes, the lowest \( M \) is achieved when the frequency spacing is \( \Delta f = 100 \text{ Hz} \). Since \( S = 1/\Delta f \), \( S = 0.01 \text{ seconds} \). By increasing the sampling frequency in part 4b from \( f_s = 362 \text{ kHz} \) to \( f_s = 409.6 \text{ kHz} \), we will have \( M = 4096 \text{ samples} \) (and Fourier magnitudes) in 0.01 seconds.

4d  \( H_n \) is the Fourier coefficient at the frequency \( f_n = n 100 \text{ Hz} \)

\[
\frac{V_{out}}{V_{in}} = \frac{1}{H_0} \sqrt{\left[ \text{Re}(H_n) \right]^2 + \left[ \text{Im}(H_n) \right]^2}
\]

\( H_0 = \frac{\sin(\pi \mu \text{s } f_n)}{(\pi \mu \text{s } f_n)} \)

Note 1: The gain is computed as the output amplitude (Fourier magnitude) divided by the input magnitude of the 1 \( \mu \text{s} \) pulses at that frequency. The response of the Butterworth anti-aliasing filter does not enter because its gain is >0.999 below 100 kHz.

Note 2: The gain is normalized to 1 at zero frequency
**Midterm #2 class statistics:**

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Grade distribution:

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6 A’s; 5 B’s; 4 C’s