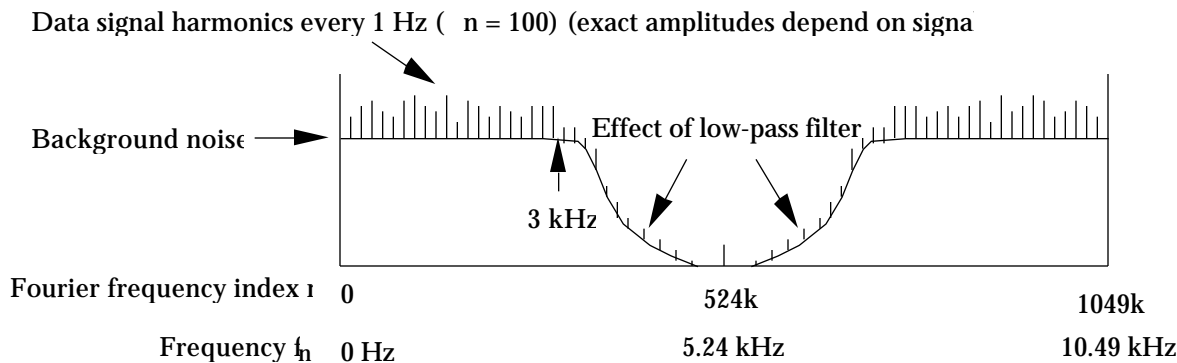


UNIVERSITY OF CALIFORNIA  
College of Engineering  
Electrical Engineering and Computer Sciences Department  
145M Microcomputer Interfacing Lab  
Final Exam Solutions      May 19, 2000

- 1a** Aliasing occurs if the waveform contains frequencies above one-half the sampling frequency.
- 1b** The Fourier Frequency Convolution Theorem states that multiplication in the time domain is equivalent to convolution in the frequency domain. Sampling a waveform  $h(t)$  at frequency  $f_s$  is equivalent to multiplication by a sum of delta functions with a time spacing  $1/f_s$ . This in turn is equivalent to convolving the full frequency spectrum of  $h(t)$  by a sum of delta functions with a frequency spacing  $f_s$ . So if  $h(t)$  is not bandwidth limited between  $-f_s/2$  and  $+f_s/2$ , sampling at  $1/f_s$  will cause overlap in the frequency domain. This causes the frequency band between  $-f_s/2$  and  $+f_s/2$  to be contaminated by higher frequencies.  
[5 points off for only stating the theorem] [5 points off for only stating that  $h(t)$  is multiplied by a sum of delta functions.]
- 1c** (1) Increase  $f_s$  to be twice the highest frequency present in the signal. This spreads out the delta functions that are convolved with the Fourier transform of  $h(t)$  and eliminates the overlap in the frequency domain.  
(2) Use a low pass filter to eliminate all frequencies above  $f_s/2$ . This removes the higher frequencies that would otherwise overlap lower frequencies.
- 2a** Spectral leakage occurs in a Fourier transform when any frequency component is not sampled for an integer number of cycles.
- 2b** Sampling a waveform  $h(t)$  from time  $-T/2$  to time  $+T/2$  is the same as multiplying by a rectangular time window  $r(t)$  that has value one between  $-T/2$  and  $+T/2$  and value zero elsewhere. The Fourier transform of the product  $h(t) \times r(t)$  is the Fourier transform of  $h(t)$  convolved with the Fourier transform of the rectangle  $r(t)$ . Since the Fourier transform of a rectangle falls off slowly with frequency  $[\sin(\pi f T)/(\pi f T)]$ , the Fourier transform of  $h(t) \times r(t)$  is a smeared version of the Fourier transform of  $h(t)$ .
- 2c** Multiply the sampled waveform  $h(t)$  by a Hanning window  $w(t)$ . Since the Fourier transform of  $w(t)$  has a small amplitude away from  $f = 0$ , the Fourier transform of  $h(t) \times w(t)$  has little spectral leakage.
- 3a** The frequency index extends from  $n = 0$  to  $n = 2^{20} = 1049k$ . The highest frequency has index  $524k$  and corresponds to the Nyquist limit of 5 kHz. The amplitude spectrum (below) has three major components
- (1) The harmonics of the data signal appear at  $n = k \cdot 100$  (where  $k$  is the harmonic number), because the fundamental has 100 cycles in the 100 s sampling window. [4 points off if omitted]
- (2) A white noise background that adds equally into all Fourier amplitudes. (Actually, the FFT of white noise is itself noisy, but we ignore that complication.) [4 points off if omitted]

(3) The effect of the low pass filter, which decreases all Fourier amplitudes sharply from 2,000 Hz (the maximum frequency of interest) to 5 kHz (the Nyquist limit). [4 points off if omitted]



**3b** The first Fourier magnitude  $F_n$  corresponds to  $f_n = n/100$  s =  $n$  (0.01 Hz).

**3c** Design requirements: Gain  $> G_1 = 0.99$  for  $f < f_1 = 3,000$  Hz; Gain  $< G_2 = 0.001$  for all frequencies that could alias to 3,000 Hz.

Choose  $n = 8$ ; from table  $f_1/f_c = 0.784$ ;  $f_c = 3,000$  Hz/ $0.784 = 3,827$  Hz;  $f_2/f_c = 2.371$ ;  $f_2 = 9,074$  Hz; since  $f_2$  aliases to  $f_s - f_2 = f_1$ , then  $f_s > f_1 + f_2 = 12,074$  Hz, which exceeds our sampling frequency of 10,486 Hz.

Choose  $n = 10$ ; from table  $f_1/f_c = 0.823$ ;  $f_c = 3$  kHz/ $0.823 = 3.645$  kHz;  $f_2/f_c = 1.995$ ;  $f_2 = 7.272$  kHz; since  $f_2$  aliases to  $f_s - f_2 = f_1$ , then  $f_s > f_1 + f_2 = 10.272$  kHz, which is just a bit below our sampling frequency of 10,486 Hz.

[Both  $n = 10$  and  $n = 12$  were accepted]

**3d** There is no spectral leakage for the data signal, since exactly 100 periods are present. Spectral leakage does not affect the random background noise since there is no structure to its frequency spectrum. Therefore a Hanning window would not help.

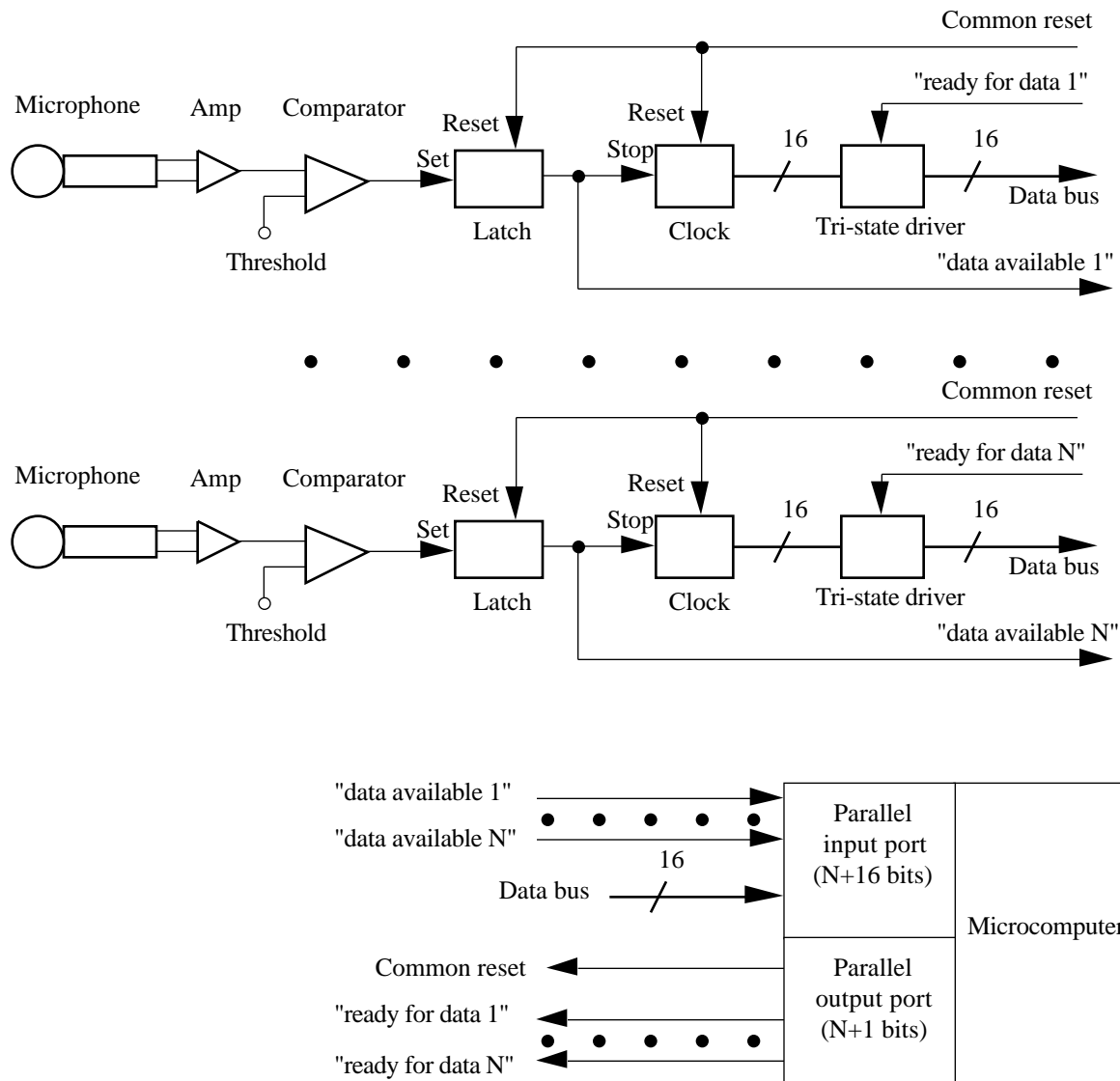
**3e** Program steps:

- 1 Sample for 100 s at 10,486 Hz (1,048,600 values).
- 2 take the FFT to compute the complex Fourier amplitudes  $F_k$
- 3 estimate the background noise frequency amplitude  $B$  as the average of amplitudes whose Fourier frequency indices are not a multiple of 100.
- 4 compute 10,486 new Fourier amplitudes  $P_k = F_{100k} - B$
- 5 take the inverse FFT of  $P_k$  to recover one cycle of the analog data signal
- 6 demodulate to recover the digital signal

[5 points off if background is subtracted from  $2^{20}$  Fourier coefficients and only one second of the inverse FFT is used]

Note: this problem works out better if the signal were repeated for a number of seconds that is a power of two (e.g. 128 seconds).

4a



4b Hardware and software steps:

- 1 The central computer resets all timers, set/reset latches, and "ready for data" lines. This starts all timers simultaneously and resets all "data available" lines.
- 2 The central computer listens for any "data available" signal from the remote sensing stations
- 3 When a microphone at a remote sensing station detects a sound that exceeds a preset comparator threshold, the corresponding set/reset latch is set to stop the counter and signal "data available"
- 4 When a "data available" signal is detected, the central computer enables the corresponding tri-state driver by asserting "ready for data" to put the counter data on the bus

- 5 The central computer uses its parallel input port to read the data from the bus and stores it in a memory location for that sensing station
- 6 The central computer resets “ready for data” (essential to put the tri-state driver into high impedance mode)
- 7 Loop back to step 2 to collect data from any other remote sensing stations that heard the gunshot. If enough time has passed to allow all stations to report the gunshot (a second or two at most), go to the next step.
- 8 Use the recorded times to compute the map coordinates and transmit to police cars
- 9 Loop back to step 1 to prepare to record the next gunshot

**145M Final Exam Grades** (15 undergraduate students, no graduate students):

|         |      |      |      |      |       |
|---------|------|------|------|------|-------|
| Problem | 1    | 2    | 3    | 4    | Total |
| Average | 32.3 | 32.3 | 58.7 | 46.9 | 170.2 |
| rms     | 6.7  | 4.1  | 5.3  | 5.0  | 13.0  |
| Maximum | 40   | 40   | 70   | 50   | 200   |

**145M Numerical Grades** (15 undergraduate students, no graduate students):

|         | Lab total | Lab Partic. | Midterm #1 | Midterm #2 | Final | <b>Total</b> |
|---------|-----------|-------------|------------|------------|-------|--------------|
| Average | 426.2     | 100.0       | 92.8       | 78.0       | 170.2 | <b>867.2</b> |
| rms     | 10.6      | 0.0         | 4.3        | 9.6        | 13.0  | <b>25.1</b>  |
| Maximum | 450       | 100         | 100        | 100        | 200   | <b>950</b>   |

**145M Letter Grade Distribution**

| Letter Grade | Course Totals (1000 max) |
|--------------|--------------------------|
| A+           | 911                      |
| A            | 907, 899                 |
| A-           | 888                      |
| B+           | 877, 873, 870            |
| B            | 865, 860, 853            |
| B-           | 844, 843, 842, 839, 837  |
| C+           |                          |
| C            |                          |
| C-           |                          |
| D+           |                          |
| D            |                          |