The first stage provides high input impedance, the second stage is a summing amplifier that also inverts the sum, and the final stage re-inverts the signal. The minimal solution requires five resistors and four op-amps.

[20 points off for drawing an instrumentation amplifier, which is designed to take the difference between two signals, not add them]
[5 points off if inputs are not high impedance]
[5 points off if no final inversion after the summing amplifier]
[5 points off for not relating the coefficients a and b to the elements in your circuit]
[3 points off for labeling amplifier gains a and b but not explicitly relating them to the resistor values]
[2 points off for omitting the series resistor in the final inverting amplifier- this connects an op-amp output directly to a virtual ground.
[5 students out of 25 got a perfect score in this problem]

2
• Connect a harmonic generator to both inputs and measure both the input amplitude $V_1(f)$ and the output amplitude $V_0(f)$ for the ten frequencies.
• Compute the common mode gain $G_C(f) = V_0(f)/V_1(f)$
• Leave the generator connected to the + input and ground the – input.
• Measure both the input amplitude $V_1(f)$ and the output amplitude $V_0(f)$ for the ten frequencies
  $V_0(f) = G_C(f)V_1(f)/2 + G_{±}(f)V_1(f)$
• Compute $G_{±}(f) = V_0(f)/V_1(f) - G_C(f)/2$
[4 points off for ignoring the common mode output when determining the differential gain.]
[3 points off for using $V_2(t) = -V_1(t)$ without providing a circuit for generating these signals]
[15 students out of 25 got a perfect score in this problem]

3
Frequency ratio $f_2/f_1 = 60 \text{ kHz}/20 \text{ kHz} = 3$
This means that the gain must drop from $G_1 = 0.999$ to $G_2 = 0.0001$ in a factor of 3 in frequency
For $n = 10$ the frequency ratio for $G_2$ and $G_1 = 2.512/0.733 = 3.43$ (too large a frequency ratio)
For $n = 12$, the frequency ratio = $2.154/0.772 = 2.79$ (okay)
$60 \text{ kHz}/2.154 < f_c < 20 \text{ kHz}/0.772$
The answer is $n = 12$ and $27.9 \text{ kHz} < f_c < 25.9 \text{ kHz}$
[14 students out of 25 got a perfect score in this problem]
4a The electromagnetic isolation amplifier
1) uses the input signal to modulate a higher-frequency carrier wave in the input stage
2) couples the modulated carrier signal to the output stage by electromagnetic induction through the air (this blocks dangerous low frequency voltages)
3) demodulates and amplifies the modulated carrier signal in the output stage to recover an isolated, amplified version of the input signal
[6 points off for describing the isolation transformer without the carrier, or differential amplification to remove electromagnetic interference, or analog filtering]
[7 students out of 25 got a perfect score in this problem]

4b The digital angle encoder
1) Has digital codes arranged in circular rings on a disk
2) Has sensors to read the digital codes
3) When the disk is rotated, the sensors pick up digital signals that are uniquely related to the angle
[4 points off if the sensors are not described]
[8 students out of 25 got a perfect score in this problem]

4c The stepping motor
1) Has a permanent magnet with alternating N and S poles that rotates within a circular array of electromagnets
2) Current in the electromagnets causes the permanent magnet to hold its position
3) Changing the currents in the electromagnets causes the permanent magnet to rotate
[18 students out of 25 got a perfect score in this problem]

5 \[ R_T = R_1 + R_2, \] where \( R_1 \) and \( R_2 \) are independently random 1 K\( \Omega \) resistors
\[ \sigma_T^2 = \left( \frac{\partial R_T}{\partial R_1} \right)^2 \sigma_1^2 + \left( \frac{\partial R_T}{\partial R_2} \right)^2 \sigma_2^2 = \sigma_1^2 + \sigma_2^2 \]
Since \( \sigma_1 = \sigma_2 \), we have \( \sigma_T = \sqrt{2} \sigma_1 \)
So the average resistance of the “2 k\( \Omega \)” resistors is 2100 \( \Omega \) and the standard deviation is 141 \( \Omega \)
[15 students out of 25 got a perfect score in this problem]

145L midterm #1 grade distribution:

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<th>Problem</th>
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<th>average score</th>
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