1a

\[ V_0 = aV_+ + bV_- \]

\[ V_0 = G_s (V_+ - V_-) + G_C (V_+ + V_-)/2 \]

\[ aV_+ + bV_- = (G_s + G_C/2) V_+ + (-G_s + G_C/2) V_- \]

\[ a = G_s + G_C/2 \]

\[ b = -G_s + G_C/2 \]

Adding, \( G_C = a + b \)

Subtracting, \( G_s = (a - b)/2 \)

Alternative solution:

Since the common mode gain is the change in \( V_0 \) per unit change in \( (V_+ + V_-)/2 \), we can add 1 V to both \( V_+ \) and \( V_- \) and see that \( \Delta V_0 = a + b \). So \( G_C = a + b \)

Since the differential gain is the change in \( V_0 \) per unit change in \( (V_+ - V_-) \), we can add 0.5 V to \( V_+ \), subtract 0.5 V from \( V_- \) and see that \( \Delta V_0 = (a/2 - b)/2 \). So \( G_s = (a - b)/2 \)

2a

<table>
<thead>
<tr>
<th></th>
<th>Op Amp</th>
<th>Inverting op-amp circuit amplifier</th>
<th>Non-inverting op-amp circuit amplifier</th>
<th>Differential op-amp circuit amplifier</th>
<th>Instrumentation amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>High ( Z_{in} )</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Differential input</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Defined gain over a frequency band</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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</tbody>
</table>

[1 point off for each wrong answer]

3a

At 10 Hz, \( A = 10^5 \) and the op-amp equation gives \( V_3 = -V_0/10^5 \) (virtual ground)

\[ (V_1 - V_2)/100 \text{ k}\Omega - V_2/1 \text{ k}\Omega - V_2/1 \text{ k}\Omega = 0 \]

\[ V_1 - V_2 - 200V_2 = 0 \]

\[ V_2 = V_1/201 \]

\[ V_2/1 \text{ k}\Omega + V_0/100 \text{ k}\Omega = 0 \]

\[ 100 V_2 + V_0 = 0 \]

\[ V_0 = -100 V_2 = -0.5 V_0 \]

\[ V_3 = -0.5 \times 10^{-5} V_1 \quad (\sim 0 \text{ was also accepted}) \]

3b

At 1 MHz, \( A = 1 \), the op-amp equation gives \( V_0 = -V_3 \)

\[ (V_1 - V_2)/100 \text{ k}\Omega + (V_3 - V_2)/1 \text{ k}\Omega - V_2/1 \text{ k}\Omega = 0 \]

\[ V_1 - V_2 + 100V_3 - 100V_2 - 100 V_2 = 0 \]

\[ V_1 + 100V_3 - 201V_2 = 0 \]
(V_2 - V_3)/1 \, k\Omega + (V_0 - V_3)/100 \, k\Omega = 0
100V_2 - 100V_3 + V_0 - V_3 = 0
100V_2 - 102V_3 = 0
V_2 \approx V_3

V_1 \approx 100V_3 \approx 100V_2 \approx -100 \, V_0
V_3 \approx V_1/100
V_2 \approx V_1/100
V_0 \approx -V_1/100

**Alternative solution: solve for any value of \( A \) and plug in for 10 Hz and 10^6 Hz**

Op-amp equation \( V_0 = -AV_3 \)

Kirchhoff’s current law at node \( V_2 \):

\[
\frac{V_1 - V_2}{100 \, k\Omega} + \frac{V_3 - V_2}{1 \, k\Omega} + \frac{0 - V_2}{1 \, k\Omega} = 0
\]

\[
V_1 = V_2 + 100V_2 - 100V_3 + 100V_2 = 201V_2 - 100V_3
\]

Kirchhoff’s current law at node \( V_3 \):

\[
\frac{V_2 - V_3}{1 \, k\Omega} + \frac{V_0 - V_3}{100 \, k\Omega} = 0
\]

\[
100V_2 = 100V_3 + V_3 - V_0 = (101 + A)V_3
\]

\[
V_1 = \left[ \frac{201(101 + A) - 100}{100} \right] \quad V_3 = \left[ \frac{10301 + 201A}{100} \right] \quad V_3
\]

\[
V_2 = \left[ \frac{201 - 100(100)}{101 + A} \right] \quad V_2 = \left[ \frac{10301 + 201A}{101 + A} \right] \quad V_2
\]

\[
V_2 = \frac{(101 + A)V_1}{10301 + 201A} \approx \frac{1 + A}{100 + 2A} \quad V_1
\]

\[
V_3 = \frac{100V_1}{10301 + 201A} \approx \frac{V_1}{100 + 2A}
\]

\[
V_0 = \frac{-100AV_1}{10301 + 201A} \approx \frac{-AV_1}{100 + 2A}
\]

**f = 10 Hz, A = 10^5**

\[
V_2 \approx V_1/201 \approx 5 \times 10^{-3} \, V_1
\]

\[
V_3 \approx 100V_1/(201 \times 10^5) \approx 5 \times 10^{-6} \, V_1
\]

\[
V_0 \approx -0.5 \, V_1
\]

**f = 1 MHz, A = 1**

\[
V_2 \approx 100V_1/10000 \approx 10^{-2} \, V_1
\]

\[
V_3 \approx 100V_1/10000 \approx 10^{-2} \, V_1
\]

\[
V_0 \approx -10^{-2} \, V_1
\]
4a

[1 point off for showing a constant gain of 0.001 below 2 Hz and above 55 kHz]

4b

The LPF needs to have a gain $G_1 = 0.9$ at $f_1 = 20$ kHz and drop to a gain $G_2 < 0.001$ at $f_2 = 55$ kHz. Assuming that the corner frequency is near 20 kHz, find the smallest value of $n$ for which the gain $=0.001$ occurs at a value of $f_2/f_c$ less than $55$ kHz/$20$ kHz $= 2.75$. Looking at the LPF table, we see that $f/f_c = 3.162$ at $n = 6$, $f/f_c = 2.371$ at $n = 8$, and $f/f_c = 1.995$ at $n = 10$.

Alternatively, we can use the fact that for a LPF with $G_2 << 1$, $G_2 \approx \left(f/f_c\right)^{-n}$ and $n = -\ln(G_2)/\ln(f/f_c) = -\ln(0.001)/\ln(55/20) = 6.83$.

Choosing $n = 8$, $G_1 = 0.9$ is at $f1/fc = 20$ kHz/$fc = 0.913$, and $fc = 21,906$ Hz
$G2 = 0.001$ is at $f2/fc = 2.371$ from which we compute $f2 = 51,939$ Hz, which is less than 55 kHz as required.

LPF $n = 8$, $f_c = 21.91$ kHz

The HPF needs to have a gain $G_1 = 0.9$ at 100 Hz and drop to a gain $G_2 = 0.001$ at 2 Hz.
Assuming that the corner frequency is near 100 Hz, find the smallest value of $n$ for which the gain $=0.001$ occurs at a value of $f2/fc$ greater than $2$ Hz/$100$ Hz $= 0.02$. Looking at the HPF table, we see that $f/f_c = 0.032$ at $n = 2$ and $f/f_c = 0.178$ at $n = 4$.

Alternatively, we can use the fact that for a HPF with $G_2 << 1$, $G_2 \approx \left(f/f_c\right)^n$ and $n = \ln(G_2)/\ln(f/f_c) = \ln(0.001)/\ln(2/100) = 1.77$. 

October 5, 2005  S. Derenzo
Choosing $n = 2$, $G_1 = 0.9$ is at $f_i/f_c = 100 \text{ Hz}/f_c = 1.437$, and $f_c = 69 \text{ Hz}$

$G_2 = 0.001$ is at $f_2/f_c = 0.032$ from which we compute $f_2 = 2.2 \text{ Hz}$, which is greater than 2 Hz as required.

<table>
<thead>
<tr>
<th>HPF $n = 2$, $f_c = 69 \text{ Hz}$</th>
</tr>
</thead>
</table>

$n = 4$, $f_c = 83 \text{ Hz}$ was also accepted

The HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01. A notch filter with accurate components should provide the necessary low gain.

[3 points off for using a 10 or 12 pole HPF rather than a notch filter to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz- this uses 4 or 5 more op-amps, is inefficient, and has more components that can fail]

### 145L midterm #1 grade distribution:

<table>
<thead>
<tr>
<th>Problem</th>
<th>maximum score</th>
<th>average score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.0 (3.5 rms) (15 max)</td>
<td>65-69 0 F</td>
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<tr>
<td>2</td>
<td>14.4 (1.2 rms) (15 max)</td>
<td>70-74 1 D</td>
</tr>
<tr>
<td>3</td>
<td>32.3 (4.6 rms) (35 max)</td>
<td>75-79 2 C</td>
</tr>
<tr>
<td>4</td>
<td>31.7 (5.0 rms) (35 max)</td>
<td>80-84 1 C</td>
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<td>85-89 2 B</td>
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October 5, 2005

S. Derenzo