1a
Johnson Noise

<table>
<thead>
<tr>
<th>Increases with temperature</th>
<th>Shot noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occurs in resistors</td>
<td>Occurs in resistors and conductors</td>
</tr>
<tr>
<td>Random voltage fluctuations caused by thermal agitation of electrons in a resistor</td>
<td>Current fluctuations caused by random fluctuations in the number of electrons per unit time</td>
</tr>
</tbody>
</table>

[-2 points for each missing answer out of 4 statements]
[Several students wrote that one was white noise and the other was not. The noise power for both is proportional to $V_{\text{rms}}^2$ or $I_{\text{rms}}^2$ which is proportional to $\Delta f$. So both are white noise.]

1b
Johnson Noise

<table>
<thead>
<tr>
<th>White (random) noise</th>
<th>Electromagnetic Interference Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to thermal motion of electrons in a resistor</td>
<td>From other circuits (or lightning)</td>
</tr>
<tr>
<td>Increases with temperature</td>
<td>Does not depend on temperature</td>
</tr>
<tr>
<td>Cannot be cancelled by differential amplification</td>
<td>Can be cancelled by differential amplification</td>
</tr>
<tr>
<td>Cannot be reduced by conductive shielding</td>
<td>Can be reduced by conductive shielding</td>
</tr>
</tbody>
</table>

[-2 points for each missing answer out of 4 statements]
[Many students gave two statements that described one difference. Four statements are needed to describe two differences between two items.]

2a
Virtual short rule $V_5 = V_6$

$$\frac{V_3 - V_5}{R_3} = \frac{V_5 - V_0}{R_4} \quad \frac{V_4 - V_5}{R_3} = \frac{V_5}{R_4}$$

$$V_3 R_4 - V_5 R_4 = V_5 R_3 - V_0 R_3 \quad V_4 R_4 - V_5 R_4 = V_5 R_3$$

Subtracting, $(V_4 - V_3) R_4 = V_0 R_3$

$$V_0 = (V_4 - V_3) (R_4 / R_3)$$

2b
Differential gain $V_0 = G_\pm (V_4 - V_3) + G_C (V_4 + V_3) / 2$

$$G_\pm = \frac{R_4}{R_3} \quad \text{Since } V_0 \text{ does not depend on } (V_3 + V_4), G_C = 0$$

[-2 points for $G_C \neq 0$]
From textbook, example 2.1, page 91:
common mode \( V_- = V_+ \)

Virtual short rule \( V_- = V_2 = V_+ = V_1 \)

No current through R1 and R2: \( V_- = V_2 = V_4 = V_+ = V_1 = V_3 \)

\[
G_c = \frac{V_3 + V_4}{V_- + V_+} = 1
\]

From textbook, example 2.2, pages 91 and 92:
Virtual short rule \( V_- = V_1 \) and \( V_+ = V_2 \)

The same current flows through R1 and R2: \( \frac{V_2 - V_1}{R_1} = \frac{V_4 - V_3}{R_1 + 2R_2} \)

Differential gain of the first stage:
\[
\frac{V_4 - V_3}{V_+ - V_-} = \frac{R_1 + 2R_2}{R_1}
\]

From textbook, example 2.2, pages 91 and 92:

\[
\frac{V_0}{V_+ - V_-} = \left( \frac{V_4 - V_3}{V_+ - V_-} \right) \left( \frac{V_0}{V_4 - V_3} \right) = \left( \frac{R_1 + 2R_2}{R_1} \right) \left( \frac{R_4}{R_3} \right)
\]

\( V_3 = \frac{V_-(R_1 + R_2)}{R_1} \) \hspace{1cm} \( V_4 = \frac{V_+(R_1 + R_2)}{R_1} \)

\[
\frac{V_4 + V_3}{V_+ + V_-} = \frac{R_1 + R_2}{R_1}
\]

\( \frac{V_4 - V_3}{V_+ - V_-} = \frac{R_1 + R_2}{R_1} \)

The new design is inferior because its first stage common mode gain is as high as the differential gain and prone to saturation. The standard design has a first stage common mode gain of only one.
[2 points for not producing an output that varied from 0 to 10 V as the liquid level varied from 0 m to 10 m]

[-4 points off for not providing a buffer amplifier between the 10 kΩ sensor resistor and the readout circuit]

**5b** Determining the change in the liquid level per minute requires making two measurements one minute apart and taking the difference. The standard deviation of each measurement is 1 mV, which corresponds to 1 mm in liquid level.

\[
f = \frac{(a - b)}{\Delta T} \quad \sigma_a = \sigma_b = 1 \text{ mm} \quad \sigma_f^2 = (\sigma_a^2 + \sigma_b^2) / (\Delta T)^2 = 2 \text{ (mm / min)}^2
\]

\[
\sigma_f = 1.414 \text{ mm / min} \quad (1.4 \text{ mm was accepted for full credit})
\]

[-3 points for 1 mV] [-1 point for 1.4 mV] [-3 points for 2 mV]

[-2 points for 1 mm] [-2 points for 2 mm]

**Note 1:** The equation sheet said that if \( f = k(a - b) \) then \( \sigma_f^2 = k^2(\sigma_a^2 + \sigma_b^2) \)

**Note 2:** The rate of change in liquid level is measured in mm/min, not mV.

6

**High pass filter**

\( f_c = 1 \text{ Hz} \quad n = 1 \)

**Low pass filter**

\( f_c = 10,000 \text{ Hz} \quad n = 2 \)

[-1 points for HPF with \( n = 2 \)] [-2 points for HPF with \( n > 2 \)]

[-1 points for LPF with \( n = 3 \)] [-2 points for LPF with \( n > 3 \)]

**145L midterm #1 grade distribution:**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Problem mark (rms) (max mark)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.6 (4.7 rms) (16 max)</td>
</tr>
<tr>
<td>2</td>
<td>19.2 (2.0 rms) (20 max)</td>
</tr>
<tr>
<td>3</td>
<td>15.8 (5.5 rms) (20 max)</td>
</tr>
<tr>
<td>4</td>
<td>15.2 (6.1 rms) (20 max)</td>
</tr>
<tr>
<td>5</td>
<td>8.8 (4.8 rms) (16 max)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Problem</th>
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</tr>
</thead>
<tbody>
<tr>
<td>30-39</td>
<td>F</td>
</tr>
<tr>
<td>40-49</td>
<td>F</td>
</tr>
<tr>
<td>50-59</td>
<td>D</td>
</tr>
<tr>
<td>60-69</td>
<td>C</td>
</tr>
<tr>
<td>70-79</td>
<td>B-</td>
</tr>
<tr>
<td>80-89</td>
<td>B</td>
</tr>
<tr>
<td>90-97</td>
<td>A</td>
</tr>
</tbody>
</table>

Maximum score = 100
Average score = 76.8 (17.1 rms)

October 6, 2004
Midterm #1 Solutions – EECS 145L Fall 2004

6 6.2 (1.5 rms) (16 max) 98 (max) 1 A+

3 graduate students: average = 79.7
8 BioEngineering undergraduates: average = 78.6
13 EECS undergraduates: average = 73.3