Midterm #1 Solutions – EECS 145L Fall 2002

1a The op-amp equation is $V_0 = A(V_+ - V_-)$

If V_0 is finite and A is infinite, then $V_+ = V_-$ (virtual short rule)

Since no current flows in or out of the op-amp inputs

$$\frac{V_1 - V_-}{R_1} - \frac{V_- - V_0}{R_2} = 0 \qquad \frac{V_2 - V_+}{R_1} - \frac{V_+}{R_2} = 0$$

$$V_1 R_2 - V_- R_2 = V_- R_1 - V_0 R_1 \qquad V_2 R_2 - V_+ R_2 = V_+ R_1$$

$$V_-(R_1 + R_2) = V_0 R_1 + V_1 R_2 \qquad V_+(R_1 + R_2) = V_2 R_2$$

$$V_0 R_1 + V_1 R_2 = V_2 R_2 \qquad \boxed{G_\pm = V_0 / (V_2 - V_1) = R_2 / R_1}$$

[5 points off for correct setup followed by algebraic errors]

[15 points off for incorrect setup]

1b To determine common mode gain, set $V_1 = V_2$ in the equation $\frac{V_2 - V_1}{R_1} = \frac{V_0}{R_2}$, and we have $V_0 = 0$. Since $V_0 = G_\pm (V_2 - V_1) + Gc (V_1 + V_2)/2$ and $V_0 = 0$ for all $V_1 = V_2$, then $G_c = 0$.

[5 points off for $G_c = V_0/(V_2 - V_1)$]

2a Gain-bandwidth product is 100 MHz; gain is 100; bandwidth is 1 MHz.
Instrumentation amplifier input noise rms = 12.9 nV x sqrt(1 MHz) = 12.9 µV
Resistor input noise rms = 128.7 µV (from equation sheet)
Total output noise = 100 x sqrt ($128.7^2 + 12.9^2$) µV = 12.93 mV.
[1 point off for simple addition of noise contributions- same for 2b, 2c, 2d, and 2e]
[1 point off for not multiplying by the gain to get *output* voltage- same for 2b, 2c, 2d, and 2e]

2b Gain-bandwidth product is 100 MHz; gain is 10,000; bandwidth is 10 kHz.
Instrumentation amplifier input noise rms = 12.9 nV x sqrt(10 kHz) = 1.29 µV
Resistor input noise rms = 12.87 µV
Total output noise = 10,000 x sqrt ($12.87^2 + 1.29^2$) µV = 129.3 mV.

2c Gain-bandwidth product is 100 MHz; gain is 10,000; bandwidth is 10 kHz.
Instrumentation amplifier input noise rms = 12.9 nV x sqrt(10 kHz) = 1.29 µV
Resistor input noise rms = 12.87 µV / sqrt(300/75) = 6.45 µV
Total output noise = 10,000 x sqrt ($6.45^2 + 1.29^2$) µV = 65.8 mV.

2d Gain-bandwidth product is 100 MHz; gain is 10,000; bandwidth is 10 kHz.
Instrumentation amplifier input noise rms = 12.9 nV x sqrt(10 kHz) = 1.29 µV
Resistor input noise rms = 12.87 µV / sqrt(1 MΩ/ 500 kΩ) = 9.10 µV
Total output noise = 10,000 x sqrt ($9.10^2 + 1.29^2$) µV = 91.9 mV.

2e Gain-bandwidth product is 100 MHz; gain is 10,000; bandwidth is 10 kHz.
Instrumentation amplifier input noise rms = 12.9 nV x sqrt(10 kHz) = 1.29 µV
Single 500 kΩ resistor input noise rms = 9.10 µV (from 2d)
Total output noise = 10,000 x sqrt ($9.10^2 + 9.10^2 + 1.29^2$) µV = 129.3 mV.

[Note: Since the rms noise is proportional to the square root of the resistance, the noise from two R/2 resistors in series must be added in quadrature to equal the noise from a single resistor R. Simple addition of noise would make two R/2 resistors in series noisier than a single resistor R]

2f T = 300 K $(12.9/128.7)^2$ = 3 K
(i.e. To reduce the resistor noise by a factor of 10 from 12.9 µV to 1.29 µV, it is necessary to reduce the temperature by a factor of 100)

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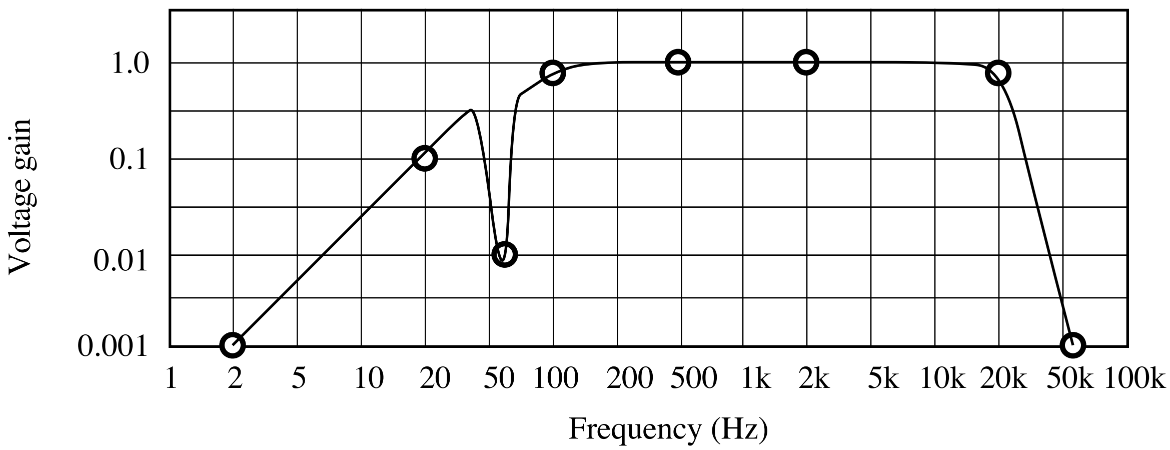
3a



3b

The LPF needs to have a gain $G_1 = 0.9$ at $f_1 = 20$ kHz and drop to a gain $G_2 = 0.001$ at $f_2 = 52$ kHz.

n	f_1/f_c	f_{1c}	f_2/f_c	f_{2c}	
4	0.834	23,981	5.623	9,248	n too low
6	0.886	22,573	3.162	16,445	n too low
8	0.913	21,906	2.371	21,932	n = 8 OK
10	0.930	21,505	1.995	26,065	n high, but OK

LPF n = 8, $f_c = 21.91$ kHz

[3 points off for $f_c = 20$ kHz, which would make the gain 0.707 (too low) at 20 kHz]

The HPF needs to have a gain $G_1 = 0.9$ at 100 Hz and drop to a gain $G_2 = 0.001$ at 2 Hz.

n	f_1/f_c	f_{1c}	f_2/f_c	f_{2c}	
2	1.437	69.6	0.032	62.5	n = 2 OK
4	1.199	83.4	0.178	11.2	n = 4 high, but OK

HPF n = 2, $f_c = 65$ Hz

[3 points off for $f_c = 100$ Hz, which would make the gain 0.707 (too low) at 100 Hz]

The HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01. A notch filter with accurate components should provide the necessary low gain.

[3 points off for using a 10 or 12 pole HPF rather than a notch filter to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz- this uses 4 or 5 more op-amps, is inefficient, and has more components that can fail]

145L midterm #1 grade distribution:

Problem	
1	30.6 (6.2 rms) (35 max)
2	20.1 (7.4 rms) (30 max)
3	29.2 (9.8 rms) (35 max)

maximum score = 100

average score = 79.9 (rms = 19.4)

30-39	2	F
40-49	1	D
50-59	0	C-
60-69	3	C
70-79	3	B-
80-89	6	B
90-99	7	A
100	3	A+