EECS 144 & 244, Fall 2010	Lee, Roychowdhury, Seshia

Midterm 2

November 22, 2010

YOUR NAME:

Instructions:

This exam is open-book, open-notes. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 80 minutes. There are **4** questions worth a total of **160** points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one.

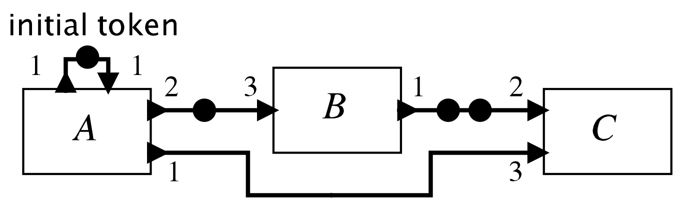
Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page. *You can use without proof any result proved in class but clearly state the result you are using.*

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: (40 points)

Consider the dataflow graph shown below:



The production and consumption parameters are shown next to each port, and initial tokens on each connection are shown with dots.

1. Write down the balance equations in matrix form.

2. Is the graph consistent? If so, give the least positive integer solution.

3. Use symbolic execution to construct an acyclic precedence graph.

4. Assume that the execution times are all one time unit. Construct the minimum makespan schedule for one iteration of the minimal balanced schedule. What is the makespan? How many processors are required to achieve this makespan?

Problem 2: (40 points)

Consider the Boolean function F of three variables x_1, x_2, x_3 that evaluates to 1 if exactly two of the variables are 1 and evaluates to 0 otherwise.

1. Draw the reduced, ordered binary decision diagram (BDD) representing this function. State the number of non-terminal nodes in the BDD.

   Does the size of the BDD depend on the variable order you select? Why or why not?

2. Write down the Boolean function in conjunctive normal form (CNF) using only variables x_1, x_2, x_3. Justify the correctness of your answer.

3. Suppose F was a function of *four* variables x_1, x_2, x_3, x_4 rather than three. Its defintion stays the same: it evaluates to 1 if exactly two of the variables are 1 and evaluates to 0 otherwise.

   How many CNF clauses (at least) would you need to write F only using variables x_1, x_2, x_3, x_4? Write the CNF representation down.

Problem 3: (40 points)

1. Let p and q be atomic propositions. Suppose that system M satisfies the LTL properties $\mathbf{G}\,\mathbf{F}\,p$ and $\mathbf{F}\,\mathbf{G}\,q$. Then, does M satisfy $\mathbf{G}\,\mathbf{F}\,(p \cdot q)$? Explain your answer.

2. Say you work for a CAD company Cadopsys, where your manager Pointy Hair has come up with an idea for performing sequential equivalence checking of two circuits C_1 and C_2, and wants you to implement it. For each circuit, the output is the entire state of that circuit. The idea goes as follows:

   - First, check that C_1 and C_2 generate the same output in the initial state.
   - Separately compute the set of reachable states R_1 of C_1 and R_2 of C_2. Store R_1 and R_2 as BDDs.
   - If $R_1 = R_2$, conclude that C_1 and C_2 are equivalent. Otherwise, report that they are not equivalent.

   Will Pointy Hair's idea work? Justify your answer.

Problem 4: (40 points)

Figure 1 shows an RC tree with N segments. $u(t)$ is an input voltage source; $R = C = 1$.

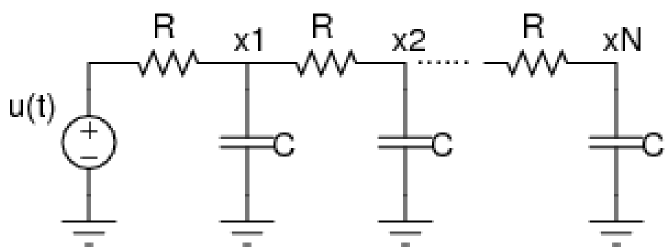


Figure 1: A simple RC circuit. (Problem 4)

1. Write out differential equations for the circuit in the matrix form $C\frac{d\vec{x}}{dt}+G\vec{x}+Bu = 0$, with the unknown variables being $x_1, x_2, \cdots, x_N$. (10 points)

2. Let $N = 2$, *i.e.*, let there be only two stages. Let the output be the node voltage at node 2. Derive the expression of the transfer function $H(j\omega) = \frac{X_2(j\omega)}{U(j\omega)}$.[1] (10 points)

[1] You may find the equality $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ useful.

3. Suppose the input signal is $u(t) = 10 + \sin(t) + \sin(2t)$. Solve for $x_2(t)$ when the circuit settles to its periodic steady state. (In your answer, you may leave $\arctan(\cdot)$ or $\tan^{-1}(\cdot)$ expressions uncalculated.) (20 points)