Midterm I - Solutions

Name: ________________________________

SID: ________________________________

Grad/Undergrad: _____________________

Closed book. One sheet of notes is allowed.
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<td>Electronic charge</td>
<td>$q$</td>
<td>$1.602 \times 10^{-19}$ C</td>
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<tr>
<td>Permittivity of vacuum</td>
<td>$\varepsilon_0$</td>
<td>$8.845 \times 10^{-14}$ F cm$^{-1}$</td>
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<td>Relative permittivity of silicon</td>
<td>$\varepsilon_{Si}/\varepsilon_0$</td>
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<td>Boltzmann’s constant</td>
<td>$k$</td>
<td>$8.617 \times 10^{-5}$ eV/ K or $1.38 \times 10^{-23}$ J K$^{-1}$</td>
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<td>Thermal voltage at $T = 300$K</td>
<td>$kT/q$</td>
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<tr>
<td>Effective density of states</td>
<td>$N_e$</td>
<td>$2.8 \times 10^{19}$ cm$^{-3}$</td>
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<tr>
<td>Effective density of states</td>
<td>$N_v$</td>
<td>$1.04 \times 10^{19}$ cm$^{-3}$</td>
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Problem 1 [15pts]

[5pts] (a) Draw qualitatively the Fermi-Dirac distribution at $T=300K$, $T=150K$, and $T\approx0K$. 

![Fermi-Dirac distribution diagram](image)
Quantum wells are often used in applications such as semiconductor lasers. In a quantum well, electrons are confined in a thin slab of material, as shown below.

The density-of-states, $D(E)$ in an ideal quantum-well is step-like:

At $T \approx 0K$, there are $1.4 \times 10^5$ states in this system, mark the location of Fermi Level, $E_f$? (In this problem, there is no need to consider spin explicitly since it is already taken into account in $D(E)$.)

At 0K, all the states below $E_f$ are filled with electrons; all the states above $E_f$ are empty. Therefore,

$$N = V \cdot n = V \cdot \int_{E_c}^{\infty} f(E) D(E) dE = V \cdot \int_{E_c}^{E_f} D(E) dE$$

This means the number of electrons in the system, $N$, equals the area under the $D(E)$ curve up to $E_f$, times the volume of the semiconductor, $V$.

$$V = 10nm \times 1000nm \times 1000nm = 10^9 \text{ nm}^3.$$

To determine the location of $E_f$, we divide the system in two regions and calculates the number of states in these regions.

1. $0.005eV < E < 0.02eV$ \quad $0.4 \times (0.02 - 0.005) \times 10^7 = 60000(\text{states})$
2. $0.02eV < E < 0.045eV$ \quad $0.8 \times (0.045 - 0.02) \times 10^7 = 200000(\text{states})$

60000 electrons will completely fill region (1); 80000 are left for region (2). Region (2) will only be partially filled. The precise location of $E_f$:

$$(E_f - 0.02eV) \times 0.8 \times 10^7 = 80000$$

$E_f = 0.03eV$
Problem 2 [30pts]

[10pts] (a) A P-N junction is fabricated by the process sequence shown below. The cartoons show cross sectional views of the wafer after each processing step. Fill in the names of each process steps shown below:

1. (Starting Wafer) 5. __etching__________.

2. __thermal oxidation________. 6. __resist strip__________.

3. photoresist spin-coating 7. __ion implantation__________.

4. __lithography__________. 8. Diffusion at 1100C for 60min

Process Available: ion implantation, sputtering, resist strip, etching, lithography, chemical mechanical polishing, epitaxy, thermal oxidation
[5pts] **(b)** The starting wafer is P-type and doped with $N_A=10^{15}\text{cm}^{-3}$. Find the depletion region width. (You may assume $N_d \gg N_a$ and $\Phi_{bi}=0.85\text{ V}$)

The implanted N-type region is usually much more heavily doped than the background P-doping ($10^{15}\text{cm}^{-3}$), this is why we can assume $N_d \gg N_a$.

We may use the lightly-doped side doping concentration to calculate $W_{dep}$.

$$W_{dep} = \sqrt{\frac{2\varepsilon_s \Phi_{bi}}{qN_a}} = \sqrt{\frac{2 \times 11.8 \times 8.854 \times 10^{14} \text{F/cm} \times 0.85\text{V}}{1.6 \times 10^{-19} \text{C} \times 10^{15} \text{cm}^{-3}}} = 1.05 \times 10^{-4} \text{cm} = 1.05\mu\text{m}$$

[5pts] **(c)** Find the depletion region width when a reverse bias of $V_R = 4.15\text{V}$ is applied. (Also assume $N_d \gg N_a$)

The depletion region is widened under reverse bias.

$$W_{dep} = \sqrt{\frac{2\varepsilon_s (\Phi_{bi} + V_R)}{qN_a}} = \sqrt{\frac{2 \times 11.8 \times 8.854 \times 10^{14} \text{F/cm} \times 5\text{V}}{1.6 \times 10^{-19} \text{C} \times 10^{15} \text{cm}^{-3}}} = 2.56 \times 10^{-4} \text{cm} = 2.56\mu\text{m}$$

[10pts] **(d)** If the diode area is $0.01\text{cm}^2$, plot $\frac{1}{C_{dep}}$ versus $V_R$. What are the x-intercept and the slope of the curve, respectively?

The depletion capacitance can be calculated using the parallel-plate capacitor formula with plate separation $W_{dep}$.

$$C_{dep} = \frac{\varepsilon_s A}{W_{dep}} = \sqrt{\frac{qN_d \varepsilon_s A^2}{2(\Phi_{bi} + V_R)}} \quad \text{(Here we also assume } N_d \gg N_a)$$

Arrange terms:

$$\frac{1}{C_{dep}^2} = \frac{2(\Phi_{bi} + V_R)}{qN_d \varepsilon_s A^2}$$

x-intercept $= -\Phi_{bi} = -0.85$

slope:

$$\frac{2}{qN_d \varepsilon_s A^2} = \frac{11.8 \times 8.854 \times 10^{-14} \text{F/cm} \times (0.01\text{cm}^2)^2}{1.6 \times 10^{-19} \text{C} \times 10^{15} \text{cm}^{-3} \times 11.8 \times 8.854 \times 10^{-14} \text{F/cm} \times (0.01\text{cm}^2)^2} = 1.20 \times 10^{20}$$

Increasing reverse bias

$$\frac{1}{F \cdot C} = 1.20 \times 10^{20} \frac{1}{F^2 \cdot V}$$

Capacitance data
Problem 3. [35pts]

The carrier profiles in a forward biased PN diode are shown below. The majority carrier densities for both P and N-side are given. Also shown is the minority carrier density on the P-side.

\[ n = 10^{17} \quad (\text{cm}^{-3}) \]

\[ p = 10^{16} \]

\[ n^\prime_p(x) \]

\[ 0 \]

\[ x \]

[5pts] (a) Calculate the bias voltage.

*From the above graph you can find the doping density.*

\[ N_d = n = 10^{17} \text{cm}^{-3}, \quad N_a = p = 10^{16} \text{cm}^{-3} \]

*From the minority carrier profile,*

\[ n^\prime_p(0) = \left( \frac{n_i^2}{N_a} \right) \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] = 10^{13} \text{cm}^{-3} \]

\[ \therefore V = \left( \frac{kT}{q} \right) \ln \left( \frac{n^\prime_p(0)}{n_i^2 / N_a} + 1 \right) = (0.026V) \ln \left( \frac{10^{13}}{(10^{10})^2 / (10^{16})} + 1 \right) = 0.539 \text{V} \]
[5pts] (b) Find the minority electron and hole current density \( J_{np}(0) \) and \( J_{pn}(0) \). Use \( D_p = 12 \text{cm}^2/\text{s}, D_n = 35 \text{cm}^2/\text{s}, \tau_p = 0.01 \mu \text{s} \) and \( \tau_n = 0.02 \mu \text{s} \).

\[
J_{np}(0) = q \frac{D_n}{L_n} \left( \frac{n_i^2}{N_a} \right) \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] = q \frac{D_n}{\sqrt{D_n \tau_n}} n'(0) = 6.69 \times 10^{-2} \text{A/cm}^2
\]

\[
J_{pn}(0) = q \frac{D_p}{\sqrt{D_p \tau_p}} \left( \frac{n_i^2}{N_d} \right) \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] = 5.54 \times 10^{-3} \text{A/cm}^2
\]

[5pts] (c) Add the minority hole concentration profile on the N-side and show \( p'_N(0) \).

\[
p'_N(0) = \left( \frac{n_i^2}{N_d} \right) \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] = 10^{12} \text{cm}^{-3}
\]
[5pts] (d) Redraw $n_p(x)$ if the electron diffusion constant $D_n$ is 4 times larger. Assume all other conditions are the same.

If the diffusion constant $D_n$ is increased by a factor of 4, the diffusion length is doubled. So the minority carrier decay length will double.

[5pts] (e) Redraw $n_p(x)$ if $5 \times 10^{15}\text{cm}^{-3}$ donors are added to the P-side and show $n_p'(0)$.
Assume all other conditions are the same.

$$n_p'(0) = \left( \frac{n_i^2}{N_a - N_d} \right) \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] = 2 \times 10^{13}\text{cm}^{-3}$$
[5pts] (f) Redraw \( n_p(x) \) the applied bias voltage is reduced by 60mV (= 0.06V).
Assume all other conditions are the same.

With a 60mV reduction the minority carrier concentration is reduced by a factor of 10.

\[
\begin{align*}
\frac{\text{cm}^{-3}}{p} & \\
10^{12} & \\
& \\
\end{align*}
\]

\[
(\text{cm}^{-3})
\]

\[
\begin{align*}
x & \\
& \\
& \\
\end{align*}
\]

\[n'_p(0) = \left(\frac{n_i^2}{N_a}\right) \left[ \exp \left( \frac{q(V - 60mV)}{kT} \right) - 1 \right] = 10^{12} \text{cm}^{-3}\]

[5pts] (h) Draw the excess minority carrier distribution if the diode is reverse biased at \( \sim 1V \).

With a reverse bias the excess minority carrier concentration at the edge of the depletion region will be

\[
\begin{align*}
\frac{\text{cm}^{-3}}{N} & \\
& \\
\frac{\text{cm}^{-3}}{p} & \\
& \\
\end{align*}
\]

\[
(\text{cm}^{-3})
\]

\[
\begin{align*}
x & \\
& \\
& \\
\end{align*}
\]

\[
\begin{align*}
-p_{N0} = -10^3 & \\
-n_{p0} = -10^4 & \\
& \\
& \\
\end{align*}
\]

\[
(\text{cm}^{-3})
\]

\[
(\text{cm}^{-3})
\]

\[
(\text{cm}^{-3})
\]

\[
(\text{cm}^{-3})
\]

\[
(\text{cm}^{-3})
\]

\[
(\text{cm}^{-3})
\]

\[
\begin{align*}
n'_p(0) = \left(\frac{n_i^2}{N_a}\right) \left[ \exp \left( \frac{q(-1V)}{kT} \right) - 1 \right] & \approx \left(\frac{n_i^2}{N_a}\right) = -n_{p0} = -10^4 \text{cm}^{-3} \\
p'_N(0) = \left(\frac{n_i^2}{N_d}\right) \left[ \exp \left( \frac{q(-1V)}{kT} \right) - 1 \right] & \approx \left(\frac{n_i^2}{N_d}\right) = -p_{N0} = -10^3 \text{cm}^{-3}
\end{align*}
\]
Problem 4. [20pts]

You are given an N-type piece of silicon with
\[ N_d(x) = (10^{16} \text{ cm}^{-3}) + (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \cdot x. \]
Assume \( T = 300 K \) and there are no acceptor dopants, i.e. \( N_a = 0 \).

[5pts] (a) Given that \( E_f \) is constant, what is the electron current density \( J_n(x) \)?

When the Fermi level is constant, the system is in equilibrium. So there will be no current flowing. The diffusion current and drift current will cancel each other out.

\[ J_n(x) = J_{n,\text{drift}}(x) + J_{n,\text{diff}}(x) = 0 \]

[5pts] (b) Find the electron diffusion current density \( J_{n,\text{diff}}(x) \). Use \( D_n = 30 \text{ cm}^2/\text{s} \).

\[
J_{n,\text{drift}}(x) = qD_n \frac{dn}{dx}
\]

\[
= qD_n \frac{d}{dx} [(10^{16} \text{ cm}^{-3}) + (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \cdot x]
\]

\[
= qD_n \times (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m})
\]

\[
= (1.6 \times 10^{-19} \text{C}) \times (30 \text{ cm}^2/\text{s}) \times (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \times (10^4 \mu\text{m}/\text{cm})
\]

\[
= 4.32 \times 10^3 \text{ A/cm}^2 \quad \text{(or} \quad 0.432 \text{ A/cm} \cdot \mu\text{m})
\]
[5pts] (c) Derive an expression for the electron drift current density \( J_{\text{drift}}(x) \) in terms of \( \mu_n \).

\[
J_{\text{drift}}(x) = q\mu_n n(x)E(x)
\]

Using Poisson’s equation and \( n(x) = N_c \exp\left(-\frac{(E_c-E_f)}{kT}\right) \)

\[
\begin{align*}
E(x) &= -\frac{d\phi}{dx} = \frac{1}{q} \frac{dE_c(x)}{dx} = \frac{1}{q} \frac{d}{dx} \left[-kT \cdot \ln \left(\frac{n(x)}{N_c}\right)\right] = -\frac{kT}{q} \cdot \frac{N_c}{n(x)} \cdot \frac{1}{N_c} \cdot \frac{dn(x)}{dx} \\
&= -\frac{kT}{q} \cdot \frac{1}{n(x)} \cdot \frac{dn(x)}{dx} = -\frac{kT}{q} \cdot \frac{9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}}{10^{16} \text{ cm}^{-3} + (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \cdot x}
\end{align*}
\]

Therefore,

\[
J_{\text{drift}}(x) = q\mu_n n(x) \left[-\frac{kT}{q} \cdot \frac{9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}}{n(x)}\right] = -kT \cdot \frac{(9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \cdot \mu_n}{n(x)}
\]

\[
= -(0.026 \text{ eV}) \cdot (1.6 \times 10^{-19} \text{ J/eV}) \cdot (9 \times 10^{16} \text{ cm}^{-3}/\mu\text{m}) \cdot (10^4 \mu\text{m/cm}) \cdot \mu_n
\]

\[
= -(3.744 \text{ C} \cdot \text{ V/cm}^4) \cdot (\mu_n \text{ cm}^2/\text{V} \cdot \text{s}) \ [\text{A/cm}^2]
\]

[2pts] (d) Using the total electron current density, find the value of \( \mu_n \).

\[
\text{From } J_n(x) = J_{\text{drift}}(x) + J_{\text{diff}}(x) = (4.32 \times 10^3) + (-3.744\mu_n) = 0,
\]

\[
\mu_n = \frac{4.32 \times 10^3 \text{ A/cm}^2}{3.744 \text{ C} \cdot \text{ V/cm}^4} = 1.154 \times 10^3 \text{ cm}^2/\text{V} \cdot \text{s}
\]

[3pts] (e) Numerically verify Einstein’s relationship with the values you’ve calculated above.

\[
\frac{D_n}{\mu_n} = \frac{30 \text{ cm}^2/\text{s}}{1.154 \times 10^3 \text{ cm}^2/\text{V} \cdot \text{s}} = 0.026 \text{ V} = \frac{kT}{q} \bigg|_{T=300K}
\]